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## CONTENTS

**Benjamin Franklin and the Mysterious "Dr. Spence"** ..... I. BERNARD COHEN 1

**Report of the Work of the Biochemical Research Foundation of The Franklin Institute.** ELLICE MCDONALD 27

**Electric Propagation on Long Lines Terminated by Lumped Networks.** MICHEL G. MALTI AND MICHAEL GOLOMB 41

**Notes from the National Bureau of Standards** ..... 75

**The Franklin Institute Notes** ..... 83

**Notes from the Biochemical Research Foundation** ..... 94

**Book Reviews** ..... 99

**Publications Received** ..... 100

**Current Topics** ..... 26, 40, 74, 82

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Journal
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The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 235 JANUARY, 1943 No. 1

# BENJAMIN FRANKLIN AND THE MYSTERIOUS "DR. SPENCE."

## The Date and Source of Franklin's Interest in Electricity.*

BY

I. BERNARD COHEN,
Department of Physics, Harvard University.

## I.

Benjamin Franklin will always be remembered for his electrical discoveries, even should the future tend to denigrate or obscure his political accomplishments. Whenever we think or speak about electricity, we do so in the words introduced into the language of electricity for the first time by Franklin. The commonest electrical words—plus, minus, positive, negative, electric motor, electrical battery, and many others—were used in an electrical sense for the first time in Franklin's writings, and the popularity of his book, "Experiments and

* I gratefully acknowledge the kindness of many friends and fellow researchers who have contributed information, criticism, and suggestions which have gone into this article: Mr. Carl Van Doren, Mr. Barney Chesnick of the Library Company of Philadelphia, Dr. Clifford K. Shipton of the American Antiquarian Society and Custodian of the Harvard University Archives, Professor Henry Guerlac of the University of Wisconsin, Mrs. Gertrude D. Hess of the American Philosophical Society Library, Professor Perry Miller of Harvard University, Professor Edward Potts Cheyney of the University of Pennsylvania, Mr. Frederick B. Tolles of Swarthmore College, and Professor and Mrs. Carl Bridenbaugh of Brown University.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

Observations on Electricity, made at Philadelphia," established them as standard for all time to come.[1]

Concerning Franklin's introduction to the science which made him internationally famous, we know practically nothing save what Franklin himself tells us. Until now, Franklin's meager statements have constituted the whole of our knowledge concerning the man who first showed him a demonstration of the "new wonders" of electricity. These statements are three in number. The first occurs in the "Autobiography," where we are told:

> In 1746, being at Boston, I met there with a Dr. Spence, who was lately arrived from Scotland, and show'd me some electric experiments. They were imperfectly perform'd, as he was not very expert; but, being on a subject quite new to me, they equally surpris'd and pleas'd me. Soon after my return to Philadelphia, our library company received from Mr. P. Collinson, Fellow of the Royal Society of London, a present of a glass tube, with some account of the use of it in making such experiments. I eagerly seized the opportunity of repeating what I had seen at Boston; and, by much practice, acquir'd great readiness in performing those, also, of which we had an account of from England, adding a number of new ones.[2]

The second also occurs in the "Autobiography," and forms the only other reference to Dr. Spence which occurs in Franklin's writings. This time we are told:

> When I disengaged myself . . . from private business, I flattered myself that, by the sufficient tho' moderate fortune I had acquir'd, I had secured leisure during the rest of my life for philosophical studies and amusements. I purchas'd all Dr. Spence's apparatus, who had come from England to lecture here, and I proceeded in my electrical experiments with great alacrity.[3]

---

[1] The extreme popularity of Franklin's book on electricity may most easily be demonstrated by the number of editions—five in English, three in French, one in German, one in Italian. This book, which in the eighteenth century was printed in ten editions in four languages, has been reprinted under the title: "Benjamin Franklin's Experiments, A New Edition of Franklin's 'Experiments and Observations on Electricity,'" edited with an introduction by I. Bernard Cohen (Cambridge, Harvard University Press, 1941). This—the eleventh edition of the book—is, oddly enough, the first to be printed in America.

[2] *The Writings of Benjamin Franklin*, Collected and Edited with a Life and Introduction by Albert Henry Smyth (10 vols.; New York, The Macmillan Co., 1905–07), vol. I, p. 417.

[3] *Ibid.*, p. 374.

Most scholars have assumed that Franklin met "Dr. Spence" first in Boston and later in Philadelphia, at which time he purchased the Doctor's "apparatus."

A comparison of these two extracts reveals a slight contradiction. The first declares that "Dr. Spence" came from Scotland, the second that he came from England. Further suspicion as to the reliability of Franklin's statement is aroused by a letter which he wrote to Peter Collinson's [4] son Michael upon the death of the latter's father. This letter reads in part:

> Our dear departed Friend, Mr. Peter Collinson . . . transmitted to the Directors (of the Library Company) . . . every philosophical Discovery; among which, in 1745, he sent over an account of the new German Experiments in Electricity, together with a Glass Tube, and some Directions for using it, so as to repeat those Experiments. This was the first Notice I had of that curious Subject, which I afterwards prosecuted with some Diligence, being encouraged by the friendly Reception he gave to the Letters I wrote him upon it.[5]

According to this letter, Collinson's gift was "the first Notice" which Franklin had of "that curious Subject" electricity.

The portion of the letter just quoted contradicts the extract quoted earlier from the "Autobiography" with regard to the date of Collinson's gift. The letter to Michael Collinson would place the date of this gift at 1745, while the first extract quoted above from the "Autobiography" would place the date at 1746.

Hence, the historian is faced with a double question. Did Franklin see "Dr. Spence" demonstrate before he received Collinson's gift? Did Franklin encounter electricity for the first time in 1745 or in 1746?

---

[4] Peter Collinson played a very important part in Franklin's scientific career. He sent Franklin (through the Library Company of Philadelphia) a gift of an "electric tube" for making experiments. To him, Franklin addressed his early letters on electricity, which contained the account of most of his fundamental discoveries. Collinson introduced these letters into the meetings of the Royal Society and eventually caused them to be printed in book form. Full details concerning the relations between Franklin and Collinson may be found in "Benjamin Franklin's Experiments" (cited in footnote 1 *supra*), intro., ch. III.

[5] Franklin to Michael Collinson, 1768 or 1769. "Writings of B. F." (*ed. cit.*), vol. V, pp. 185–186.

In the following pages it will be seen that Franklin saw "Dr. Spence" before he received Collinson's gift; in fact, he saw "Dr. Spence" in Boston in 1743 and later in Philadelphia in 1744. Incidentally, it will be shown that Franklin's "Dr. Spence" is Dr. Adam Spencer. The latter fact explains why historians who sought for information concerning "Dr. Spence" have been hitherto unsuccessful.

## II.

An advertisement which appeared in the *Boston Evening-Post*, no. 408, May 30, 1743, contained an announcement that "Dr. Spencer having a compleat Apparatus, proposes to begin a Course of Experimental Philosophy in Boston. . . ." This advertisement was repeated in the issue of June 6, 1743, and also appeared in the *Boston Weekly Post-Boy*, no. 441, May 30, 1743:

*Doctor Spencer having a compleat Apparatus, proposes to begin a Course of* Experimental Philosophy *in Boston, as soon as* Twenty *shall have subscribed (of which Notice shall be given) to be continued at such Times as shall be agreed upon by the Subscribers at the first Lecture. The charge of going through the Course is* Six Pounds, *Old Tenor, to be paid one Half at Subscribing. Those that are inclined to attend, are desired to enter their Names, and pay the Subscription Money to Mr.* Thomas Kilby *at the Naval Office, who will furnish a Catalogue of the Experiments, gratis.*

According to the terms announced by Spencer, the "Course" would be given only if twenty subscribers would enroll. In that event, Dr. Spencer declared, the time and place of the first meeting would be published in the newspapers. Since no such announcement was made, we may suppose that the cost, "Six Pounds, Old Tenor," was too great a price for the Bostonions to pay for Spencer's scientific lectures.

Some time later, Spencer appeared in Philadelphia, according to an advertisement which appeared in Franklin's *Pennsylvania Gazette*, no. 802, April 26, 1744. This reads as follows:

A Greater Number of Gentlemen having subscribed to *Dr. Spencer's* first Course of EXPERIMENTAL PHILOSOPHY, than can be con-

veniently accommodated at a Time: He begins his first Lecture of the second Course, on Thursday, the tenth day of May, at five o'Clock: Subscriptions are taken in at the Post Office, where a Catalogue of the Experiments may be had gratis.

Franklin was at that time Postmaster of Philadelphia and the Post Office was in his house at Market Street. Since subscriptions to the lectures were "taken in at the Post Office," where one could obtain a "Catalogue of the Experiments," we can be certain that Franklin knew of these lectures. In effect, he acted as Spencer's agent.

Franklin frequently referred to his business establishment simply as "the Post Office." For example, in the *Pennsylvania Gazette*, no. 798, March 29, 1744, Franklin published the following notice:

A COLLECTION of choice and valuable BOOKS, consisting of near 600 Volumes in most Faculties and Sciences, viz. *Divinity*, *History*, *Law*, *Mathematicks*, *Philosophy*, *Physick*, *Poetry*, &c. Will begin to be sold by Benjamin Franklin, for ready Money only, on Wednesday the 11th of April 1744 at Nine o'Clock in the Morning, at the Post-Office in Philadelphia; the lowest Price being for Dispatch marked in each Book: Catalogues may be had *gratis*, at the Place of Sale.

Note. *The said Franklin, gives ready Money for any Library or Parcel of Books.*

Spencer's announcement which is quoted above was reprinted in the *Pennsylvania Gazette* of the week following—no. 803, May 3, 1744. One may assume that Spencer was quite popular in Philadelphia since he felt the need of a second set of lectures. And, as if that were not sufficient, we find in the *Gazette*, no. 815, July 26, 1744, the following advertisement:

*A COURSE OF EXPERIMENTAL PHILOSOPHY, begins at the Library-Room, next Monday at five o'Clock in the Afternoon, which will be the last to be performed in this City by Dr.* Spencer.

N.B. *Any of the Gentlemen who subscribed to the former Courses, may go through this, at half Price, and have as an Addition some* Lectures *on the* Globes.

Thus far we know that a Dr. Spencer offered a course in Experimental Philosophy in Boston in 1743, but did not obtain a sufficient number of subscribers to warrant his be-

ginning the lectures. He came to Philadelphia in the spring of 1744 and attained a high degree of success and had to repeat his course several times in order to satisfy the demand. In Philadelphia, Franklin acted as his agent.

## III.

In order to learn the content of Spencer's course, we must turn to other sources of information. The first of these is the journal (1744) of William Black. Here there are two accounts of Spencer's lectures. The first of these, entered under the date of May 29, 1744, reads:

> Between the hours of 3 & 4 the Governor, Commissioners, and the rest of the Company went to hear a Philosophical Lecture on the Eye, &c., by A: Spencer, M:D:, in which he endeavoured to account for the Faculties, the Nature and Diseases of that Instrument of Sight; next he proceeded to show that Fire is Diffus'd through all Space, and may be produced from all Bodies, Sparks of Fire Emitted from the Face and Hands of a Boy Suspended Horizontally, by only rubbing a Glass Tube at his Feet.[6]

From this extract we learn that Spencer's first initial was *A*.

The second entry appears under the date of June 5, 1744, and reads as follows:

> At 11 in the forenoon, with Colonel Beverley and the Gentlemen of the Levee, I went to the State House, where Doctor Spencer Entertain'd Us very Agreeably with several Philosophical Transactions, first he Prov'd and Illustrated by Experiments, Sir Isaac Newton's Theory of Light and Colours, also Several Curious Objects Shown by the Solar Microscope, together with the Circulation of the Blood, all he perform'd very much to the Satisfaction of the Spectators.[7]

These brief notes can be somewhat amplified by those which were taken by John Smith.[8]

Smith's notes are not labelled and there is no direct evidence that they were taken on hearing Dr. Spencer. But,

[6] *The Pennsylvania Magazine of History and Biography*, I (1877), p. 246.

[7] *Ibid.*, pp. 413–414.

[8] John Smith, a Quaker intellectual, was very much interested in the progress of science. Cf. Frederick B. Tolles: "A Literary Quaker: John Smith of Burlington and Philadelphia," *The Pennsylvania Magazine of History and Biography*, LXV (1941), pp. 300–333.

since Smith was in Philadelphia in 1744, and since he was interested in science, we can reasonably assume that he attended Spencer's lectures. The similarity between his notes and those taken by William Black is ample evidence that both were derived from the same lectures. Smith's notes read as follows:

Lecture 1st.

That the properties of all matter is the same Viz. Solidity, Extention, Divisibility, mobility & passiveness or Inactivity—that upon this principle the Softest body is as Solid as the hardest, And the Lightest as the heaviest. The Matter of Cork is as Solid as Lead, nor is a Diamond a Jot more Solid than Water. Solidity is a property of matter whereby it Excludes every other body from the place itself possesses. Extension is that whereby a thing is Constituted Long, Broad, or thick &Ca.:

Divisibiiity, a passive power, or property in quantity whereby it becomes separable into parts, either actually or at least mentally. The Dr. Supposes, Matter admits of a Mathematical Divisibility ad Infinitum—as in this Example $\frac{1}{4}$ one divided into four parts. If I add an o it [*the denominator*] is 40 if another it is 400 if another 4000, & so endless. Upon this head he observed that One Grain of Gold, by the Workmens Coarse Tools, could be workt into 50 square Inches—that the Length of an Inch could be Divided into 200 Visible parts, so that in one Square Inch there is 40 000 Visible parts; & in one Grain of Gold there are 2 000 000 of such parts; which Visible parts no one will deny to be farther Divisible. Again, an ounce of Silver may be Gilt with 8 Grains of Gold, wch. is afterwards drawn into a wire 13,000 foot Long. By the help of Glasses he said One may See Many ridges or Scales on the Surface of the hand, that by the Microscope one may Count 500 pores in one of those Scales, & one Grain of Sand will Cover 200 Scales—By the microscope many small Animals may be perceived A thousand of wch. (says the Dr.) may lead up a Country Dance on the point of a needle. And yet these must have all the parts necessary for Life, as blood, & other Liquors: How Wonderful then must be the Subtilty of the parts which make up such fluids!

Mobility is an aptitude or facility to be moved.

Passiveness or Inactivity is that power whereby Matter Resists any Change Endeavoured to be made in its State. I.E. whereby it becomes difficult to alter its state, either of rest or motion. So that it requires the same force to stop any thing when in motion, as to put it into motion.

Upon Gravitation the Dr. Observed, that Every Body Inclines towards a Center. That all Bodies near the Earth, have a Gravity or tendency toward the Centre of the Earth —. That this power Encreases as we descend & decreases as we Ascend from the Centre of the Earth, and that in the proportion of the Squares of the Distances therefrom reciprocally; so as, for Instance, at a double distance, to have but a quarter of the force &Ca..

Two Leaden Balls, almost square & Rubbed together, stuck so close that it required a great Weight to Separate them.

Divide a Loadstone in two, & one will Attract to the North; the other to the South, also two needle points being rubbed on the same Loadstone & put in Water, the Loadstone attracted one and repelled the other.

* * *

One of the Drs. Experiments relating to Electrical Attraction was thus—

He took a Long Glass Tube, & Rubbed it Vehemently with his hand, & then held it pretty near several Pieces of Leaf brass or Gold, which put them into very brisk & Surprizing Motions. Some would leap toward the Tube, Sometimes adhere & fasten to it, settle on its Surface, and there remain Quiet: and sometimes be thrown off from it with a great force. And thus would they be alternately attracted and Repelled, for several times Successively. Sometimes, again, they would move slowly toward the tube, sometimes, would remain suspended between the tube and the Block they were first laid on; and sometimes slide along in the direction of the side of the tube, without touching it.

The Dr. Argued for a Vacuum thus. Supposing the particles of Air to be Globular. Suppose them to be what shape you please. If you Admit that the Air is Composed of Particles—there must be some distance between each. Consequently a Vacuum. Again. If a small parcel of Air can be Rarified so as to Occupy a Larger Space than it naturally would do, the Vacuitys or distance between the particles must be greater. That the Air is Compounded of Particles you must allow because it has the properties of Matter as Solidity, Mobility &Ca. and you cannot Suppose Matter without particles.

A Glass of Malt Beer being put under a Receiver & the Air Exhausted, the Beer immediately set to working, & after working a while being taken out it tasted Flat & Dead.

A little Cool Air is proper in a fever but no Cordials.

All Bodies to which fire are applyed, become bigger, Swell'd and Rarifyed.—Spirit contracts in cold weather a 50th part & Expands in hot.

A Glass Ball with Rum & one with Water made thus the Ball being put into hot Water the Rum rose in y^e^. Tube much faster & higher than the water.

A Tin Vessel filled with Oyl over a fire will melt before the Oyl will boyl.

Large Bodies have always a lesser Surface in proportion to their matter than small ones.

Organs of Sence Uncertain Judges of Heat. E. Gr. That Air after a Storm in August w^ch^. we Suppose Cool is really Warm.

Heat puts the Particles in Perpetual Motion. Cold Inclines them to rest & Quiet. A Globe with a Retorted pipe the end of the pipe being put in Cold Water & the Globe in hot (such a one as this ) The heat Rarifying the Inclosed Air caused it to bubble out thro' the water, & the Globe being taken out of the hot Water, the Cold arose in the Pipe.

An Experiment was made on the Pyrometer of the different degrees of Expansion of a Bar of Copper & a Bar of Iron by the same heat, & both being heated 40 degrees, the Iron Expanded 24 and the Copper 56.

* * *

Galileo Discovered the Elasticity or Spring of the Air & proved it by this Experiment. An Extraordinary Quantity of Air, being Intruded by means of a Syringe into a Glass or Metal Ball. Upon taking the Syringe away, the Air rushes out. Consequently it is Endued with an Elastic force.

Bubbles are dilatable or Compressible. I. E. take up more or less room as the Included Air is more or less heated, or more or less pressed from without; and are round because the Included Aura acts Equably from within all around. Their Coat or Cover is formed of the minute particles of the fluid, retained either by the Velocity of the Air, or by the brisk Attraction between those minute parts and the Air.

The Lower Air is always denser than the Upper: Yet the Density of the Lower Air is not proportional to the Weight of the Atmosphere by Reason of Heat & Cold which make notable Alterations as to rarity & Density.

* * *

The Light is only seven minutes passing from the Sun to us.

The Globe of the Eye Consists of 6 Coats & 3 humours. For an Inflammation in the Eye, Cupping by the side of it is Good.

All our Senses are feeling.

Oyl of Vitriol & Cold Water mixt became Immediately Warm.

Oyl of Vitriol & Oyl of Tartar mixt produced Ebulition.

Spirit of Nitre & the Oyl of Cloves mixt will produce flame.

A Bullet out of a Gun moves 1142 feet in a Second & the Strongest Wind but 35 foot & says the Dr. its friction against such a quantity of particles of Air, heats it as it goes, so that when it falls it is very warm, & the further it goes, the hotter it is.

The Strongest Winds are always attended with a hot Air.

Take the filings of Steel & Sulphur, Tempered with Water, & put under Ground will produce an Earthquake.

Clouds of Ice melting together in the Air by their Violent Attrition may cause a Lightning, & this heat Rarifying the Air, Other Air rushing in there causes Thunder.

Fire is diffused thro' all Space, Contained in, & may be produced from all bodies.

A Boy was suspended Horizontally & the Dr. rubbed a Glass Tube, a little distance from his feet wch. made Sparks of fire fly from his face & hands.[9]

Dr. Spencer evidently dispersed a few little practical medical precepts among his remarks on natural philosophy. Remarks such as: "A little Cool Air is proper in a fever but no Cordials," and "For an Inflammation of the Eye, Cupping by the side of it is good," served to alleviate the barrenness of pure science by a little bit of practical advice for the benefit of the audience.

As a whole, these notes are interesting as an example of the kind of science which was being diffused in the Colonies in the 1740's. There was a demonstration of Newton's discovery of dispersion and the nature of color, of Harvey's discovery of the circulation of the blood. We may note, too, that Spencer referred frequently to experiments and to measurements (i.e. to numerical data) and that he performed many experimental demonstrations to illustrate the topics on which he lectured.

Our attention centers quite naturally on Spencer's remarks concerning electricity. Following a discussion of the principles of solidity and infinite divisibility of matter, and of mobility and passivity, he turned to gravitational attraction

[9] These notes are published for the first time from the originals in The Library Company of Philadelphia (Ridgway Library). They may be found in Smith MSS., vol. V, pp. 254–255. They cover both sides of two sheets of paper. It is by no means certain whether these notes represent one lecture or several. Nor can one tell what order the sheets should follow. I have therefore indicated the page separation by means of asterisks.

and thence to magnetism. He showed his audience that if a lodestone is broken, the newly exposed ends of the two pieces exhibit opposite polarities. If one is South, the other is North. The concomitant attraction to magnetism is that of electricity, to which Spencer next devoted his attention.

He showed his hearers first that if a glass tube is rubbed, it will attract small pieces of metal, and that some of these, after being attracted to the extent of touching the rubbed glass, will then be repelled. Electrical attraction of this kind (due to rubbed amber) had been known to the Greeks; repulsion had been discovered in the seventeenth century by Cabeo.[10] These simple and elementary phenomena represent knowledge on such an elementary level that Franklin, in his book on electricity, took them for granted and did not bother to describe them, although until Franklin put forth his unified theory of electrical action (in terms of a single electric fluid), they were not fully understood.

The concluding item in Black's account as well as in Smith's concerns the electrification of a boy suspended horizontally from the ceiling. This experiment had been performed for the first time by Stephen Gray and announced by him on April 8, 1730.[11] Electricity became a popular subject in the middle of the eighteenth century and, of all the experiments used to demonstrate the novelty of the subject, this one was certainly one of the most widely-used. An illustration of the manner of performing this experiment appears below as Fig. 1.

Franklin himself never performed this particular demonstration so far as we know. But he did electrify human beings for more genuinely scientific purposes, such as to test the value of an electric shock as a therapeutic agent in certain types of paralysis, and he frequently electrified himself in the course of various electrostatic experiments. Rather than suspend an innocent victim from the ceiling by means of silk

[10] A convenient summary of the history of electricity prior to Franklin's research may be found in "Benjamin Franklin's Experiments" (cited in footnote 1 *supra*), intro., ch. II.

[11] Stephen Gray: "Several Experiments Concerning Electricity," *Philosophical Transactions of the Royal Society*, XXXVII (for the years 1731, 1732), p. 39.

FIG. 1. Frontispiece to the Abbé Nollet's *Essai sur l'électricité des corps* (Paris, 1746).

threads, Franklin made use of the equally effective, if less dramatic method, of having the person to be electrified stand or sit on a glass stool which insulated him equally well.

Two years after Spencer lectured in Philadelphia, the Leyden jar (first type of condenser) was invented and the whole science of electricity was speedily transformed. The new invention offered a means of "storing" an electric charge and opened up the possibility of experimenting with electricity on a large scale. Franklin's work was accomplished on the next level of electrical knowledge above that concerning which Dr. Spencer lectured in Philadelphia. The only immediate link between Franklin's writing on electricity and Dr. Spencer's lectures occurs in the penultimate sentence in Smith's notes. "Fire is diffused thro' all Space, Contained in, & may be produced from all bodies." By "fire," Spencer referred to electricity, which was then usually called "electrical fire" or simply "fire."

Two years after Franklin began the investigation of electrical phenomena he wrote his paper entitled: "Opinions and Conjectures concerning the Properties and effects of the Electrical Matter . . . Made at Philadelphia, 1749." In this paper, Franklin, for the first time, used the expression "electrical fluid" for electricity, abandoning the older expression "electrical fire." The opening paragraph of this paper of Franklin's reads:

> The electrical matter consists of particles extremely subtile, since it can permeate common matter, even the densest metals, with such ease and freedom as not to receive any perceptible resistance.

But that Franklin and Spencer should here be in accord is no occasion for surprise. Like all other sons of the eighteenth century, both were thinking in terms of an "imponderable fluid" which could pass in and out of matter with little or no resistance, i.e. a "subtile fluid."

What distinguishes Franklin's theory of electricity from that of his contemporaries and predecessors is not that he thought in terms of such a fluid, since that was a common habit of thought, but rather the particular use he made of such a fluid in establishing a unitary theory of electrical action.[12]

[12] Cf. "Benjmain Franklin's Experiments" (*op. cit.*), intro., ch. III.

## IV.

Before returning to the question of whether or not Dr. A. Spencer is the "Dr. Spence" of Franklin's "Autobiography," let us complete the account of Dr. Spencer. There are several other sources of information concerning him.

Although we have no record that Spencer lectured in Philadelphia after the July series of 1744, we know that he was still in Philadelphia during September. In the "Itinerarium" of Dr. Alexander Hamilton, we find the following extract under the year 1744:

> Friday, September 14th.—I . . . drank tea with my landlady Mrs. Cume, and at five o'clock went to the coffee-house, where I saw Dr. Spencer, who for some time had held a course of physical lectures of the experimental kind here and at New York. . . . I met here likewise one Mitchell, a practitioner of physick in Virginia, who was travelling as he told me upon account of his health.[13]

The Mitchell in question—the "practioner of physick in Virginia"—is Dr. John Mitchell, for many years resident in that state and who was famous for his botanical activities. He returned to England in 1746 and soon afterwards published his famed *Map of the British and French Dominions in North America* (London, 1755).[14]

Some time between the spring of 1743 when he was in Boston and the spring of 1744 when he was in Philadelphia, Spencer visited New York. In a letter dated January 22, 1744, James Alexander wrote as follows to Cadwallader Colden:[15]

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[13] "Hamilton's Itinerarium, Being a Narrative of a Journey from Annapolis, Maryland, through Delaware, Pennsylvania, New York, New Jersey, Connecticut, Rhode Island, Massachusetts and New Hampshire, from May to September, 1744," by Doctor Alexander Hamilton, edited by Albert Bushnell Hart (Saint Louis, 1907), pp. 232–233.

[14] It was to Mitchell, rather than to Collinson, that Franklin addressed his first paper on the electrical nature of lightning. Cf. "Benjamin Franklin's Experiments" (*op. cit.*). An account of Mitchell may be found in the "Dictionary of American Biography."

[15] James Alexander and Cadwallader Colden were amateur scientists who were friends and scientific correspondents of Franklin. Colden, Lieutenant-Governor of New York, was particularly interested in gravitation and electricity. Accounts of both appear in the *D.A.B.*

The Comet was Seen at Philadelphia before Christmas—I heard nothing of it till the 2$^{d}$ of January when D$^{r}$ Spencer told me that he was told a blazeing Star was Seen by the people. I had askt him & M$^{r}$ Kennedy to Spend the Evening, but was kept from them by Alderman Johnson Mr Murray & Mr. Smith . . . , when they departed casting up my Eyes to the Stars at first sight I saw the Comet & went in & told Mr Kennedy & D$^{r}$ Spencer I had seen it & it was very visible—as I had their Company I Did not Attempt to make any Observation of it that night.[16]

This is the same comet concerning which Franklin informed his readers throughout the winter at the end of 1743 and the beginning of 1744. One such notice, typical of the set, appeared in the *Pennsylvania Gazette*, no. 792, Feb. 16, 1744, sandwiched in between news of ship arrivals in Philadelphia and a notice concerning "the Body of one Smith" which "was found floating in the River near Point Nopoint." This notice reads:

The Comet, which has been seen in the Evening these Six Weeks past, now appears in the Morning, rising before the Sun. It is very much encresed in Bigness and Length of Tail since its first Appearance.

Approximately one year after Dr. Hamilton recorded his meeting with Dr. Spencer and Dr. Mitchell, the latter wrote a letter to Colden which is of double interest. It contains news of Spencer and it throws light on Franklin's early unsuccessful attempts to start his scientific society in Philadelphia (i.e. The American Philosophical Society). This letter is written from Mitchell's estate in Urbanna, Virginia, and is dated September 10, 1745. Concerning an essay of his, he writes:

I thought indeed that it might make a fit piece for the records of the Philosophic Society at Philadelphia; but was sorry to find none such, . . . & am still more sorry to hear, by D$^{r}$ Spencer, that their laudable design is in a manner entirely dropt.[17]

[16] "Colden Papers," vol. III, *Collections of the New York Historical Society* (for the year 1919), p. 46.

[17] "Colden Papers," vol. VIII, *Coll. of the N. Y. Hist. Soc.* (for the year 1934), p. 321.

Colden replied to Mitchell in a letter dated New York, November 7, 1745, which concluded with the words: "Please . . . to give my Service to Dr Spencer." [18]

A few further items concerning Spencer's career were uncovered by Mr. Carl Bridenbaugh in the course of his research into eighteenth-century Philadelphia. One of these occurs in Thomas Cadwalader's "An Essay on the West India Dry-Gripes . . ." (Philadelphia, B. Franklin, 1745). Mr. Bridenbaugh informs me:

> There are two copies in the College of Physicians in Philadelphia; the prefaces differ. In #2 preface Cadwalader says: (pp. iii–iv) he thanks "my learned and worthy Friend, Dr. A. Spencer, of Philadelphia, (who is justly recommended by the famous Dr. Mead, and several eminent Gentlemen of the Faculty in London, as a most judicious and experienced Physician and Man-Midwife) for his Trouble in revising this Essay, . . ."

Other material concerning Spencer, including the fact that his first name is Adam, appears in "Rebels and Gentlemen. Philadelphia in the Age of Franklin" (New York, Reynal & Hitchcock, 1942) by Carl and Jessica Bridenbaugh. Here we learn (p. 269) that Spencer was trained in Edinburgh and that he went to Newport some time between the summer of 1743 when he was in Boston and the spring of 1744 when he was in Philadelphia. In Newport he encouraged Dr. Thomas Moffatt and the ingenious clockmaker William Claggett to undertake electrical experiments.

## V

Finally we come to the question: Is Dr. A. Spencer the mysterious "Dr. Spence" of Franklin's "Autobiography"?

Franklin certainly knew of Spencer, since he published advertisements concerning his lectures in the *Pennsylvania Gazette*, acted as his agent to the extent of distributing the programs of the lectures, since he was a member of the board of directors of the Library Company of Philadelphia (in whose rooms in the State House Spencer gave his course). In order to use the rooms of the Library Company, one first

[18] *Ibid.*, p. 338.

had to obtain the permission of the directors, as we shall have occasion to note more fully in the following section. And, as if that were not sufficient, Dr. Spencer was acquainted with three of Franklin's friends and scientific correspondents —Colden, Alexander, Mitchell. There can be no doubt that Franklin knew of Spencer's activities in Philadelphia.

Any one who knows anything at all about Benjamin Franklin would find it hard to believe that a man of his curiosity and inquisitiveness and with his great interest in science would have limited his relations with Spencer to a purely commercial level. He would certainly have talked with him about scientific questions and most probably attended his lectures as well. If Dr. Adam Spencer is not "Dr. Spence" we must, in any case, revise our notions concerning the date of Franklin's introduction to electricity since Spencer was in Philadelphia in 1744 and supposedly Franklin did not encounter "Dr. Spence" until 1746.

But for the date and the final *r* to the name, the data concerning Spencer and "Spence" are identical, as may be seen as follows:

1. According to the "Autobiography," a "Dr. Spence" was in Boston in 1746 where Franklin met him. According to the "Autobiography," "Spence" showed Franklin some "electric experiments." Franklin writes that he "show'd me some electric experiments," from which we may assume that Franklin was given a private showing and did not attend a public lecture-demonstration.

According to the newspapers, Spencer was in Boston in 1743 and offered to give a course of lectures on experimental philosophy, but could not obtain a sufficient number of subscribers to warrant giving the course.

2. Later, "Dr. Spence" came to Philadelphia "to lecture" and Franklin bought "all Dr. Spence's apparatus."

In 1744, Spencer came to Philadelphia and lectured over a period of several months.

The first question to be answered is whether or not Franklin could have met Dr. Spencer in Boston in 1743. According to Mr. Carl Van Doren's recent biography, Franklin visited New England during the spring of 1743,[19] that is,

[19] Carl Van Doren: *Benjamin Franklin* (New York, Viking, 1938), p. 140.

during the time when Spencer was making his unsuccessful effort to obtain members for the projected course.

As I have pointed out earlier, Franklin's reliability on the question of the date of his introduction to electricity is questionable. He wrote that Peter Collinson's gift of an electric tube arrived soon after his return to Boston from the visit during which he saw "Spence." The "Autobiography" thus places the date of Collinson's gift at 1746 or 1747. But, in Franklin's letter to Collinson's son Michael, the date of this gift is placed at 1745. If we knew the exact date of Collinson's gift, we might have an immediate answer.

The gift was sent to the Library Company but, unfortunately, the Minute Book is very incomplete for the early period and contains no record of Collinson's gift. Both the "Autobiography" and the letter to Michael Collinson were written many years after Franklin had begun his work in electricity and he may only have remembered that some time before 1747 (when he wrote his first letter to Collinson) he had received a gift of an electric tube and that some time before the gift arrived he had seen Spencer's demonstration (or "Spence's"). Since his memory of the date was, at least, unreliable, it may just as well have been completely in error. And if he did not remember the date correctly, he may just as well have forgotten that the name was actually "Spencer" and have thought it was "Spence."

The error in the doctor's name may have been introduced by any of several other means. It may have derived from the carelessness of William Temple Franklin in the course of preparing the publication of the "Autobiography," or it may have been simply a printer's misprint.[20] One is led to wonder whether Franklin's reference to "a Dr. Spence" was not intended to read "Dr. A. Spencer." Most likely this last is not the case, and Franklin's reference to "a Dr. Spence" was couched in that term merely because he did not remember much about the Doctor—neither the person nor the date when he met him.

Plainly, then, Franklin knew Dr. Spencer and Dr. Spencer

[20] For the vagaries in the publication of Franklin's "Autobiography," see Max Farrand: "Benjamin Franklin's Memoirs," *The Huntington Library Bulletin*, no. 10 (October 1936), pp. 49–78.

could satisfy the account of "Spence" given in the "Autobiography," if we are willing to accept two very plausible sources of error, both on Franklin's part. If we are not willing to face the identification of "Dr. Spence" as Dr. A. Spencer, then we must make the unlikely supposition that two men, with almost identical names, came to America within a period of two years, that both went first to Boston (and did not give public lectures), that both then went to Philadelphia (where they did give public lectures) and both knew Franklin, that both gave lectures on electricity, but that *while one left behind him an easily traceable record, the other left behind no trace of his visit save a mention in Franklin's* "Autobiography."

But, apart from the fact that it is more reasonable to suppose that Dr. Spencer is the true "Dr. Spence," there is a last bit of evidence to support this proposition. From the lecture notes of Spencer's Philadelphia lectures, we learn that Dr. Spencer demonstrated the circulation of the blood. Since the notes would not appear to indicate that the Doctor performed a dissection, one would suppose that he accomplished his demonstration by means of a machine designed to show the action of the heart and circulatory system, a common enough device in the eighteenth century. Franklin, in his "Autobiography," does not say that he bought only the Doctor's electrical apparatus, but rather that he bought "all Dr. Spence's apparatus, who had come to lecture here." If this were Spencer, then he would have bought the apparatus for demonstrating the circulation of the blood. Franklin owned such an apparatus which he treasured as one of his prize possessions. When Manasseh Cutler visited the aged Franklin, he reported that he was shown a "glass machine for exhibiting the circulation of the blood in the arteries and veins of the human body." [21] This may, however, have been the machine which Franklin in 1745 described to Colden as "a siphon made of two large joints of Carolina cane . . . into which two small glass tubes are to be inserted . . ." [22] al-

[21] "Life, Journals and Correspondence of Rev. Manasseh Cutler," edited by William Parker Cutler and Julia Perkins Cutler (2 vols.; Cincinnati, 1888), vol. I, p. 269. Cf. Carl Van Doren: "Benjamin Franklin" (*op. cit.*), p. 750.

[22] Franklin's two letters concerning the circulation of the blood were written to Colden in 1745 and may be found in "Writings of B.F." (*op. cit.*), vol. II, pp. 284–289, 290–294.

though from the description, one would gather that this was not a permanent piece of apparatus, but rather a make-shift contrivance to serve its purpose for the time being. After 1745, Franklin never again returned to the subject of the circulation of the blood and it seems unlikely that he would have kept his temporary contrivance until he died. On the other hand, a permanent well-made apparatus of glass, such as he bought from the Doctor, was the sort of instrument which he would have kept.

To sum up briefly. The accumulated evidence would tend to show that in its form as printed, Franklin's "Autobiography" contains two errors concerning his introduction to electricity. First, the person who showed Franklin his initial demonstration of electrical phenomena was not "a Dr. Spence," but rather Dr. A. Spencer. Second, rather than in 1746, Franklin's meetings with the Doctor took place in Boston in the spring of 1743 and in Philadelphia in the spring and summer of 1744.

[After this article had been completed, Mr. Carl Van Doren called my attention to a piece of evidence which seems decisive in the reconstruction attempted above. The weak link is, clearly, whether or not Franklin did meet Dr. A. Spencer in Boston in 1743. The remaining piece of evidence occurs in "Franklin's Draft Scheme of the Autobiography," published by A. H. Smyth in vol. i of "The Writings of Benjamin Franklin." Here one finds: ". . . Success in Business. Fire Companies. Engines. *Go again to Boston in 1743. See Dr. Spence.* Whitefield. My connection with him. His generosity to me. My return. . . ." I have italicized the two strategic phrases.]

### SUPPLEMENTARY NOTE ON SCIENTIFIC LECTURES IN MID-EIGHTEENTH-CENTURY PHILADELPHIA.

Dr. A. Spencer was far from being the only lecturer on scientific subjects to visit Philadelphia during the 1740's. And Franklin's activities on his behalf were hardly an example of a unique service.

Several years prior to Spencer's visit, Franklin performed an identical service for another visiting lecturer on science, namely Isaac Greenwood, who was, incidentally, the first

native American scientist in an American university to be educated abroad. Greenwood, born in America, was graduated from Harvard College in 1721. He went to England in 1723 and returned three years later. Late in 1726, he announced series of public lectures to be given in Boston. These lectures were delivered in the first months of 1727 and dealt largely with aspects of Newtonian mechanics. They appear to have been the first public scientific lectures to be given in the Colonies. The following year he became the first Hollis Professor of Mathematics and Natural Philosophy at Harvard.

During the first four years of his tenure, he published scientific papers in the *Philosophical Transactions* of the Royal Society of London and communicated with various English scientists. He continued to give public scientific lectures in Boston during the 1730's, but his period of active scientific investigation had already ceased, a result no doubt of his interest in the "Demon Rum." In 1738 he was discharged from Harvard for continual drunkenness.

In the Minute Book of the Library Company of Philadelphia, we find the following extract under the date of May 28, 1740:

> At a Meeting of the Directors Present Israel Pemberton Jnr. I. Stretch, W. Coleman, B. Franklin, P. Syng, E. Morgan, S. Rhoads, & Ph. Bond to consider a Request made by B. Franklin in Behalf of Isaac Greenwood Professor of the Mathematics of Natural Philosophy, that they will lend him their Air-Pump, & allow him the Use of the Outer-Room adjoining to the Library, there to exhibit Mathematical & Philosophical Lectures & Experiments. The Directors willing to encourage so useful a Design, agreed to grant the said Request for such a Time as will be sufficient for going thro' Once Course of Experiments and Lectures.

Mr. Chesnick informs me that "the Minutes do not give any information as to whether or not the lectures were delivered."

One may note that Greenwood's title has become confused in this extract and rather than Professor of Mathematics and Natural Philosophy, he is called Professor of the Mathematics of Natural Philosophy. The writer of the Minutes was probably thinking in terms of Newton's great treatise, the

*Juſt arrived from* LONDON,

For the ENTERTAINMENT of the CURIOUS and OTHERS,

And is now to be SEEN, by Six or more, in a large commodious ROOM, at the Houſe of Mr. *Vidal*, in Second-Street;

*The Solar or Camera Obſcura* MICROSCOPE,

INVENTED by the Ingenious Dr. LIBERKHUN.

IT is the moſt Entertaining of any MICROSCOPE whatſoever, and magnifies Objects to a moſt ſurpriſing Degree. The Animalcule in ſeveral Sorts of Fluids, with many other living and dead Objects, too tedious to mention, will be ſhewn moſt incredibly magnified, at the ſame Time diſtinct; alſo the Circulation of the Blood in a Frog's Foot, a Fiſh's Tail, alſo in a Flea, and Louſe, where you diſcover the Pulſe of the Heart, the moving of the Bowels, the Veins and Arteries, and many ſmall Inſects, that one Thouſand of them will not exceed the Bigneſs of a Grain of Sand, with their Young in them; Eels in Paſte, which have given a general Satisfaction to all that ever ſaw them. This Curioſity was never ſhewn by any Perſon that ever travelled. Price *Eighteen Pence*.

*Each Seed includes a Plant, that Plant, again*
*Has other Seeds, which other Plants contain:*
*Thoſe other Plants have all their Seeds, and thoſe,*
*More Plants again, ſucceſſively incloſe.*
*Thus, every ſingle Berry that we find*
*Has, really, in itſelf, whole Forefts of its kind.*
*Empires and Worlds one Acorn may diſperſe*
*By Plants to fail a Thouſand Ages hence.*

*Each Myrtle Seed includes a Thouſand Groves,*
*Where future Bards may warble forth their Loves:*
*So Adam's Loins contain'd his large Poſterity,*
*All People that have been, and all that's to be.*
*Aſtoniſhing Thought! what Mortal can conceive*
*Such wond'rous Smallneſs!——yet, unwiſe believe*
*What Reaſon tells: for Reaſon (piercing Eye)*
*Diſcerns thoſe Truths our Senſes can't deſcry.*

Note, *The* MICROSCOPE *may be ſeen at Gentlemens Houſes, giving half an Hour's Notice, the Sun permitting only from Ten in the Morning to Four in the Afternoon, in my Room.*

THE unparallel'd MUSICAL CLOCK, made by that great Maſter of Machinery DAVID LOCKWOOD: This great Curioſity performs by Springs only; It is a Machine incomparable in its Kind; it excels all others in the Beauty of its Structure; is moſt Extraordinary in its Muſick; and plays the choiceſt Airs from the moſt celebrated Operas, with the greateſt Nicety and Exactneſs: It performs with beautiful Graces, ingeniouſly and variouſly intermix'd, the French Horn Pieces, perform'd upon the Organ, Germain, and Common Flute, Harpſichord, &c. as Sonata's, Concerto's, Marches, Minuets, Jiggs, and Scot Airs, compoſe'd by Corelli, Alberoni, Mr. Handel, and other great and eminent Maſters of Muſick. Price *Eighteen Pence*.

This beautiful Curioſity has been ſhewn twice before the KING, in his Royal Palace at St. James's, where His Majeſty was pleas'd to make an Obſervation on the Excellence of its Beauty, and declar'd, He thought it the Wonder of this Age. It is allowed by all who have ſeen it, to be more worthy to adorn a King's Palace than of being expos'd for a common Sight.

*N. B.* This ſurpriſing Piece of Machinery has given ſuch general Satisfaction to the Lovers of Art and Ingenuity, that the Nobility are continually commanding it to their Seats to ſhew their Curioſity; and is to be SOLD by the Owner EDWARD RUSH.

The Inſide of this Machine may be view'd by Gentlemen and Ladies, and is to be ſeen from Eight in the Morning till Eight at Night.

*For the Evening Entertainment,*

THE Clock and Camera Obſcura, with the Battle of Dunkirque, and ſeveral Indian Land-ſkips, repreſenting Armies, both Horſe and Foot, going through their Exerciſe at the Word of Command: Likewiſe Views of Ships fighting at Sea, with the Fiſh playing above Water, and Variety of Country Dances by Figures, ſix or eight Foot high, perform'd in a beautiful Manner by the Camera Obſcura. This Curioſity is eſteem'd one of the beſt Pieces of Painting of the Kind that ever was brought from *Italy*.

FOUR SEAT *Two Shillings*. Second Ditto *One Shilling*. To begin at Seven a Clock.

FIG. 2. Advertisement in the *Pennsylvania Gazette*, No. 813, July 12, 1744.

*Principia*, "Mathematical Principles of Natural Philosophy," rather than in terms of the title of the Hollis Professorship.

Franklin not only sponsored Greenwood's application, but acted as Greenwood's agent, just as he acted as Spencer's agent, and collected the subscription fees at the Post Office. We know that this is so from an advertisement which appeared in the *Pennsylvania Gazette*, no. 599, June 5, 1740, which, incidentally seems to solve the problem as to whether Greenwood actually gave his course of lectures. This advertisement reads as follows:

> The Gentlemen who have subscribed to the *Encouragement* of a *Course of Philosophical Lectures and Experiments*, to be performed by Mr. Greenwood, are desired to meet in the *Chamber* adjoining to the *Library* at the *State-House*, on *Tuesday* next, about 9 a Clock in the Morning, when it is proposed the *Course* should begin, and be continued afterwards, at such Times as the Gentlemen who are willing to attend a *Course* at other Times, in the same Place, are desired to leave their *Names* at the *Post-Office* in *Philadelphia*, where the *Conditions* thereof may be seen, and *Subscriptions* taken in.

After leaving Philadelphia, Greenwood went to Charleston, where he engaged in private tutoring until he died in 1745.[23]

During the year 1744, Spencer was not the only person to give demonstrations in Philadelphia. The *Pennsylvania Gazette*, no. 811, June 28, 1744, carried an advertisement of an exhibition of "curioisties" which read as follows:

> *Near the upper End of Second-street, two Doors above the Sign of the Ship, is to be SEEN, in*
> John Baker's *House, after Six a Clock any Evening,*
>
> Among other CURIOSITIES,
>
> EIGHT BELLS ringing truly, both round Ringing and Changes, much in Imitation of Ringing in England, with two Young Men and a Lady walking, and the Lady turning Head over Heels, like a Mountebank, and the Clock drawing a Curtain. Also there is

[23] The information concerning the life of Isaac Greenwood was given to me by Mr. Frederick G. Kilgour. Further details concerning his life may be found in the forthcoming volume (VI) of "Sibley's *Harvard Graduates*," edited by Clifford K. Shipton. (Published in May 1942.)

to be seen a Tower and Steeple, much like to that in Bristol in England, with 8 Bells and 8 Men ringing round Ringing and Changes, likewise an Image that blows a Bellows, strikes Fire, lights a Candle and fires a Gun.

The Price of Men, 4d. Women 3d. Children 2d.
The like never before in these Parts.

The exhibition of this mechanical contrivance may not be considered in the category of science, but it is worth giving in full since it places the next item in its true setting.

In the *Gazette*, no. 813, July 12, 1744, a full-page advertisement announced an elaborate set of spectacular demonstrations. This advertisement is reproduced alongside as Fig. 2. It appeared again in the issue following, no. 814, July 19, and in a reduced form it was continued in later issues. For example, issue no. 817, Aug. 19, 1744, contained a notice of this attraction which occupied a whole single column. The popularity of the attraction is attested to by the continuing advertisements.

Evidently there was much scientific and quasi-scientific entertainment in Philadelphia during 1744. Their appreciation in the hands of the Philadelphians is attested to by the continued advertisements.

But of all the lecturers on scientific subjects in Philadelphia in the middle of the eighteenth century, the most famous and the most popular was Ebenezer Kinnersley, a native Philadelphian. Kinnersley [1711–1778] was a Baptist minister who taught English and oratory at the College of Philadelphia, which has since grown into the University of Pennsylvania. Kinnersley was a co-experimenter of Franklin's and himself made several important discoveries, such as the conducting qualities of various sorts of charcoal, and invented an "electric air thermometer." [24]

Kinnersley became famous for his lectures on electricity. He prepared these under Franklin's direction and, in fact, Franklin wrote the greater part of them. Kinnersley delivered his first public lecture in Philadelphia in 1751 and, in

[24] Kinnersley's scientific achievements, his lectures, and his life are described in full in "Benjamin Franklin's Experiments" (cited in footnote 1). There too one may find most of Kinnersley's scientific writings.

the same year, went to Boston where he repeated his lectures to an enthusiastic audience. The Bostonians liked these lectures so much that he stayed on for several months repeating them. He gave the lectures again in New York City and in Newport, R. I., and then gave them in Philadelphia, on and off, for many years. In 1753, he went to Antigua, to St. John's, and there too he gave his lectures on electricity.

Kinnersley's name was thus known throughout the North Atlantic seaboard and it is even possible that during the 1750's, his fame as electrician was greater than Franklin's. But Kinnersley lectured chiefly on Franklin's discoveries, and the chief difference between his lectures and those of Greenwood and Spencer is that he lectured exclusively on electricity, they introduced the subject of electricity merely as one of a number of subjects of scientific interest. And the reason for this lies chiefly in the fact that between the time of their lectures and his, Franklin had made enough startling discoveries with Kinnersley's aid to provide materials for two full lectures on the subject of electricity.

**Galvanized Steel as a Power Conductor.**—Electric power engineers during World War I and since will recall the efforts made to replace copper as a conductor of electricity when its scarcity became imminent. Some installations were made then and since which were successful. Late information along these lines is that furnished in an article in *Electrical World*, Vol. 118, No. 12, by FREDERICK M. CRAPS, President of the Indiana Steel and Wire Co., Muncie, Indiana. One of this company's research products is an improved type of galvanized steel conductor which combines the best electrical and physical properties of all the other types of iron and steel. This new ferrous conductor is manufactured from medium-carbon steel of special analysis and is also subject to certain special process treatments during their fabrication. Distinctive properties are relatively low 60 cycle resistance, low magnetic permeability and low 60 cycle internal reactance, in combination with relatively high tensile strength, high fatigue endurance, small creep under initial sustained loads, sufficient ductility to insure appreciable elongation before rupture, and high resistance to shock and deformation. It receives certain special heat treatments at appropriate stages in the process of cold drawing to final size, which produce advantageous end effects on both electrical properties and physical properties of the conductor. Young's modulus after initial loading to maximum design tension, or equivalent pre-stressing, is a relatively high and constant figure for stranded as well as solid conductors. Stranded steel conductors have lower skin effect, lower effective resistance and lower internal reactance than solid conductors of the same material and cross section. These decreases are attributable to the subdivision of the current among the individual wires comprising the conductor, and also to the fact that the internal magnetic effect of each strand of current is confined in large degree to its own individual wire. The use of the conductors can be extended very materially by overloading them and by tolerating greater voltage drops than permitted under normal standards of service. They will stand exceptionally high temperatures without annealing or appreciable impairment of tensile strength.

R. H. O.

FIG. 1.

## REPORT OF THE WORK OF THE BIOCHEMICAL RESEARCH FOUNDATION OF THE FRANKLIN INSTITUTE.*

BY

ELLICE McDONALD, M.D.,
Director.

This is a very difficult time at which to give a report of our laboratories. It is the first since Pearl Harbor and war to our country has brought many difficulties to research such as ours which is now almost entirely upon war defense research.

The whole program of our research has been recast and rerouted since the beginning of the war in 1939. Many humanitarian projects have been shelved for duration and many new problems have been attacked. In our choice of these war problems we have been guided by two principles—first, that they should be of value to the war effort and, second, that they should be of such humanitarian value that they could be continued and developed, if need be, after the end of the war.

Research altered again by war has altered very much within my lifetime. This is not surprising because life has altered. In one generation, all standards are astray: there

* November 18, 1942.

are no charts to guide the progress of present-day affairs. All must depend upon importunism: to do the best we can until we see how things break. Our fear is that as a result of world affairs less good things will result than the moderate good we had before and yet, as scientists, we can visualize a better arrangement than ever we had before were we free to arrange for ourselves. However, this is the world, and as realists, we must take it as we find it; we cannot make it over, at least not now.

Bacon in his essay "On Innovations" has said:

> It were good that men in their Innovations would follow the example of Time itself, which, indeed, innovateth greatly, but quietly, and by degrees scarace to be perceived. . . . It is good also not to try experiments in States except the necessity be urgent, or the utility evident; and well to beware that it be the reformation that draweth on the change, and not the desire of change that pretendeth the reformation.

There may be a lesson here for those reformers who wish to "make America over" in a hurry.

Conditions have changed so in one lifetime that it is only in retrospect that we can begin to appreciate how revolutionary these changes have been. It seems as if our advances in knowledge and technical capacity have gone so far ahead of our experience in government and control that there is a lag between them that will take a number of years to catch up. Our technical advances have been great, but there is a question whether the present deterioration of government in efficiency and honor have depreciated all our efforts, so that we are lagging rather than advancing in spite of our material progress. Certainly the cynicism of the present day, Statism (for that is what it is) seems a reaction and a rebuttal of all scientific efforts toward betterment.

The forces which have changed the world have come largely of research and discovery. Events now happening would not have been possible forty years ago and are dependent upon research good or bad as the results may be.

Undoubtedly research in its passion for detail, precision and measurement has become a trifle inhuman in the sense that it has not taken into account the low level of human appreciation. Scientists fulminate and adumbrate against present conditions to scientists, a limited group who are

mostly inarticulate. The result does not touch the mass of the people who believe in headlines. Of course, one trouble with scientists is that they are trained to tell the truth, and if they are in doubt about it, they modify their statements by so many qualifications that the final summary is denuded of real value. They suffer from what a very interesting newspaper reporter once designated as "factism." Therein they differ from the politicians who see two sides of every question: the truth and the political aspect.

Scientists want such an exact statement made that the main idea is often obscured in detail: in other words, they were speaking to their fellows and, if we continue to speak to our fellow scientists, we cannot complain if our fellows (and few of them) alone digest our statements. It is our own fault that we have not impinged more on the affairs of our people to show how management could be made more useful. Minority pressure groups have usurped the place that the advice of scientists should have in affairs of government. Our own scientific societies have bowed down in obeisance immediately the central government called them into conference and have followed like sheep.

So research in spite of its great advances has lost out to the little people. Its defects were the faults of precision, exactitude and honesty and it must continue so if it is to be research, because the qualities of research are such that the last and most fatal ditch that any research man may fall into is to alter his results according to his need and away from the truth. This is so rare that in forty years of experience only one such has come under my observation and he is now "heiling" the Hitler he then despised.

The rôle of the research worker is not such as to wield immediate world influence, although his permanent influence on world affairs is greater than many of the ephemeral exploitations and explosions of the politicians which arouse in scientists a deep distaste and distrust.

However, after all that complaining, this is the world and we have to live in it. Some years ago, some of us tried to get a definition of sanity. In the group were two psychologists (whatever they are), one psychiatrist and a medical doctor. The best definition we could get was that sanity

was "the degree of contact with reality." It takes one back to the two old Quakers—"All the world's a little queer except me and thee and sometimes I think thee a little queer."

In our time the world has changed. It has changed in so many material ways that it is only by retrospect that we can appreciate the difference. The changes which have occurred can at least be listed under three heads:

1. Annihilation of distance.
2. Scale of operations.
3. Increased use of power.

These three, although very broadly stated are at least in part the factors which have determined the enormous change in life in our time. Because this change has occurred so abruptly it is a little difficult to see in perspective. It has happened to us and the quick succession of events has followed in such sequence that they have overlapped and become a continuity, rather than being as the physicists say "particulate."

In the life history of lower organisms which can be studied in their various and succeeding generations on account of their shorter life span, succeeding generations will go along an even keel of development and arrangement so that each new organism maintains the character and resemblance of the previous parents. This will continue, for what would be in terms of mankind, thousands of generations. There then sometimes comes, from whatever unknown cause, a deviation from standard type and new generations of the organisms develop to a different form from their progenitors, so that each succeeding type of organism is different from those which have gone before. This is called a "mutation period" where, in the course of evolution, the organisms, such as the drosophila or fruit fly, for example, change their type.

In human affairs, certainly in discovery, this present time seems to be a mutation period for mankind. Such stirring events have happened in one man's lifetime of three scores years and ten, that if measured in events, such a man might consider himself to have lived 300 years rather than the allotted time of the Psalmist. The revolution of life cannot but have its effect upon the nature of man, although we may

be too close to the picture to comprehend the vastness of the change. Youth takes these developments as a matter of course and it is often amusing to see the young casually accept a development or discovery which appears to those older as a new miracle.

The miracles, particularly the *annihilation of distance* which includes, indeed, ease of communication, may seem less revolutionary to those born in the more established and civilized communities than to me who was born on the frontier where the memories of youth establish a more diverse background of experience against which to weigh the great changes of modern civilization of our present time.

My early recollections were of the slow travel with its attendant discomforts and the infrequency of communications. When I have been carried in an aeroplane above the clouds at 15,000 feet and at a speed of 250 miles per hour, my recollection goes back to the early journeys and I realize that the annihilation of distance is indeed a fact. And there is now sold a beautiful box-like room on wheels which travels on the open road and the motor car makers warn not to run it at 60 miles per hour until it has been driven 400 miles. My son and a friend came in a motor from St. Joseph, Missouri, to Philadelphia—1200 odd miles without stopping to rest. There are many better record-breaking feats, but this particular one interested me because St. Joe was the outfitting place for the Caravans across the western plains before and after the gold rush of 1849, when it took six months to get there from the East and six months from there to California during which time "the banjo on my knee" must have been worn out. Yet now the journey to California can be done in one fearful leap and in about 7 hours and a bomber will fly from Britain to Canada in $9\frac{1}{4}$ hours.

An old friend, Dr. Charles Camsell, Deputy Minister of Natural Resources in Canada, was an Arctic Explorer and, in 1899–1903 he made an extensive journey on foot, and in canoes, covering 1400 miles and taking three and a half years. Thirty-five years later, he covered the same route by aeroplane with ease in ten flying days stopping overnight.

On his first journey, he discovered a tribe of Esquimaux who had never seen a white man and who had no iron: their

lances were barbed with walrus ivory: their light was from open seal oil lamps with a wick of the pith of reeds. Their knives were walrus tusks and their clothes were skins, sewn with the sinews from a caribou. They were in the stone age.

Thirty-five years later in the aeroplane, Dr. Camsell visited the same encampment and found that now they had repeating rifles, outboard motors, sewing machines, phonographs and that some of the women wore corsets. They were precipitated from the stone age to modern civilization in a few years. This transition, quick as it seems, only differs from our own by being more concise, dramatic and shorter in time, but the alteration which has taken place in our own lives is not less great nor is the variation more remarkable.

What have these things to do with research? Has research altered life? Are these the product of research or does research advance result from the accumulated discoveries of the last 50 years? As well ask, as the snow-ball rolls and grows in size with accumulation, is it due to the momentum or to the adhesive qualities of the soft snow—or is the hen before the egg which is before the hen which lays an egg.

Whatever the mechanism of the matter, there is no doubt that research in my time has been profoundly altered although some research workers do not yet see it, for, strange as it may seem, research men are sometimes most conservative and reactionary of people although their sphere should be to do with new things.

Annihilation of distance and ease of communications have profoundly influenced research. Marconi has rendered provincialism a migratory, if not a meaningless thing. Physically as a result of these discoveries, the continents are a thousand times closer together than were the scientists of a hundred years ago. The rapid dissemination of news makes us all contemporary participants in the experiences of the whole world of science. A discovery is made and it is flashed around the world to be seized upon with eagerness by all those with research curiosity. In almost no time, numbers are working, checking and developing the idea and applying it to their own problems.

If there is a new development, even a theory, men flock to the originator for instruction. In 1926, there were 20

American physicists in Göttingen where Born and Jordon announced a new development in quantum mechanics. A year later, they were at Zurich with Schrödinger, a couple of years later with Heisenberg at Leipsic: then Dirac at Cambridge was the attraction. The scientist like the bee as Horace says will sip honey from any man's hive. A personal journey to hear Bohr in 1919 gave me a sentence which altered my entire view of biological research. Bohr then had only the fixed model of the atom and he said, "From now on the face of science will be changed. We can now study from the minute to the mass instead of from the mass to the minute." It struck me that I had been studying the processes of life from the mass to the minute, downward from the branches of the tree instead of from the roots.

Men's minds are quickened and their imagination stimulated so that thought is released from the shackles of authority and dogma. Quickness of communication with the size of operations combine to accelerate the progress of research to an extraordinary degree. There are so many more people working in such larger coördinated groups that scientific research must advance by leaps and bounds. Coördination of the sciences, physics, chemistry and biology, make it possible to gain a comprehensive view of a problem where the isolated methods of the past would give no answer, but only come to an impasse.

*Size of operations* applies not only to research groups, but to the scientific societies which actually exist for the exchange of information. The big aggregations of incorporated capital is one example of the increase of size of operation and they by coöperation have made possible advances based on research which have revolutionized human affairs. The dread spectre of war itself has enlarged the hideousness of its countenance by the masses it employs.

In our laboratories, with its trained people now 50 in number and numbering more scientific people than almost any college is one example of increase in the scale of operations. Not only are these greater in numbers than most scientific people of universities; but they are free from the burden of teaching and all coördinated and organized toward the special

end without the loose medieval university organization where each professor is a satrap unto himself.

This type of organization makes possible direction of research toward particular projects and brings the whole group of scientists of varied character and apparatus of unique quality to bear on the elucidation of any problem.

Not the least is the accumulation of literature now obtainable in libraries such as ours which is now one of the best in the world in biochemical, biological and medical problems. The background of such a mass of literature of the achievements of other scientists is a modern advantage such as never before has existed and gives hope of enormous advances in the future.

*Increase in use of power*, the third category, has its prototype in science in the vast number of instruments of precision which have been devised to aid scientific observation. The number of these would require pages to enumerate and every day brings another. This increase in the power of man gives him tools to extend the value of his bodily senses as power produced by heat or electricity increases man's muscular power.

For example, our cyclotron, one of the biggest tools, weighing about 100 tons to study the smallest known particles, is an affair in development of yesterday, so new that we have not yet plumbed all its possibilities for research. Or our battery of Beams air-driven centrifuges of various forms, some of which run at 3,000 revolutions per second (not minute) and produce sedimentary forces hundreds of times that of gravity, but require an organized and arranged set-up and men with experienced knowledge to run them. Or our battery of Tiselius electrophoretic apparatus for measurement and fractionation of proteins or other substances which cannot be separated or studied by any other means. The value of the last alone in bacterial and immunological research is incalculable and brings hope of new attacks upon problems which affect life.

In our laboratories then we have a group of scientists, organized, educated and experienced upon the most advanced lines with apparatus which is unique in the world. More than that there are a closely working group of special scientists

each coöperating with the other and each bringing help on and adding new vision to any problem from the knowledge of their special phase of education.

Organizations such as this cannot be built up in a day nor can apparatus such as is in our laboratories be collected under years. It is a difficult and slow process to get the proper building with the special requirements, to collect and set up the most advanced apparatus and, the most difficult of all, to get the proper scientific people to man the ship. And then one thing more, to teach them all to coöperate with each other and to learn the joy of aiding others in their problems often of a phase of science remote from their own. It is the function of a director to know that success depends upon having a clear idea as to the objectives to attain and a definite plan as to the best way of reaching them. After that, success depends upon the gift of choosing the right persons in every branch of the research, upon his power of drawing the best service out of them by encouragement, stimulation and support; upon the exercise of such a close and constant supervision over every detail (without getting lost in a jungle of details) as will enable him to discover how things are going at all times.

Such should be the organization of a modern laboratory made up of a group working together in unison with the best tools and free and happy to carry out their chosen work. The day of the lone worker, the prima donna type, is over, although there are sure to be exceptions to every rule for genius will not be confined.

Our laboratory was well organized until the outbreak of the war which brought rearrangement in three different directions, (1) rerouting and rearranging projects so that the work would be more applicable to the war effort, (2) necessity of substituting workers for those leaving for the war efforts and (3) the shortage of materials necessary for research due to the demands of the war production.

In war time research, only immediately useful discoveries are wanted as war cannot go on forever and must be won quickly; no long-time project is permissible. To re-adapt a large research program in a group is not an easy matter. The workers themselves resent being harked off a problem

which interests them and upon which they have spent considerable time, thought and effort until they thoroughly understand the exigencies and the need.

However, three days after the outbreak of war between Germany and Great Britain, the readaptation of our program was begun because it seemed certain that war would soon touch this country. It was thought that the United States would be at war in June 1941 and plans were made on that basis, but the six months' error gave some leeway.

On the onset of war in 1939, I wrote to the Director of Medical Research in Great Britain asking him what were the most important problems to be studied. He answered that owing to this "phoney" war, he did not know. However, we immediately began to study the treatment of infected war wounds and developed a new germicide which I still believe is the best treatment for *infected* wounds.

Since then we have adapted and re-arranged our work so that we are almost entirely on problems akin to war conditions. Most of it cannot be told for the silly conditions of secrecy imposed do not permit. The "secret" discoveries so banned are often found already published in the British journals or published in some of the American journals by some Government employee. Suffice to say that our laboratories are trying to do their bit and that such contracts as we have for research work are taken without any cost whatever to the United States Government and in this we differ from the Universities. It would be interesting to know from the Office of Scientific Research and Development how many contracts are made without cost to the United States.

However, we can tell some interesting phases of our work. One of these shows the value of a group study of different scientists, particularly physico-chemical scientists, as applied to a bacterial problem, in this case typhoid.

There are many suggestions in bacteriological literature that a large part of the material injected in vaccines may be useless for the purpose of bringing about immunity. That is to say, probably only a small part of the whole bacterial body is really useful in this respect, while the remainder may be not only without effect, but also toxic or the cause of many of the bad reactions which follow vaccination. Our purpose in

the study of the typhoid organism was to see if a small part of the whole organism could be isolated which would have the useful vaccinating power of the whole organism without its toxicity. With this purpose in view, we grew large amounts of these organisms, and then made a study of the fluid medium in which they had grown, since it is known that during growth parts of the bacilli go into solution. This material was concentrated and freed from extraneous matter, and physicochemical methods applied to it as follows: It was first centrifuged in a Beam's type air-driven centrifuge for many hours at speeds upwards to 100,000 times gravity. This treatment resulted in the separation of a layer of large molecular weight which could be removed, and which by serological means was found to contain by far the largest part of the activity.

Electrophoresis was then carried out in a standard Tiselius equipment, and was found to yield a fast-migrating constituent and a group of slow-migrating constituents. These were then separated, again freed from salts and dried in the Cryovac apparatus at the temperature of dry ice. We now had highly purified fractions of a type not used before in studies on the typhoid bacillus, and with them we were able to carry out experiments both on the serology and on the immunizing and toxic powers. Of the ten fractions developed from various strains of the typhoid organism, only one was found to be toxic to mice in amounts of one milligram. The others were without effect. It is interesting to note that these fractions all gave reactions indicative of polysaccharides and did not give the reactions of proteins. In other words, we have isolated polysaccharide-containing portions, which have been shown in other organisms at least to be of decisive importance in immunity.

These portions apparently constitute the outer covering of the typhoid organisms, and normally serve to protect the bacteria against the defense mechanisms of the body. These surface antigens govern the immunological properties of the bacteria and apparently serve as the only antigenically useful portion of the vaccine. Antibodies produced against the surface antigens enable the white blood cells to engulf invading

organisms, and thus to protect the body against these pathogenic bacteria.

This discovery has far-reaching possibilities in the prevention of typhoid without the disadvantages and dangers of the present type of immunizing vaccine and it is the result of group coöperative work.

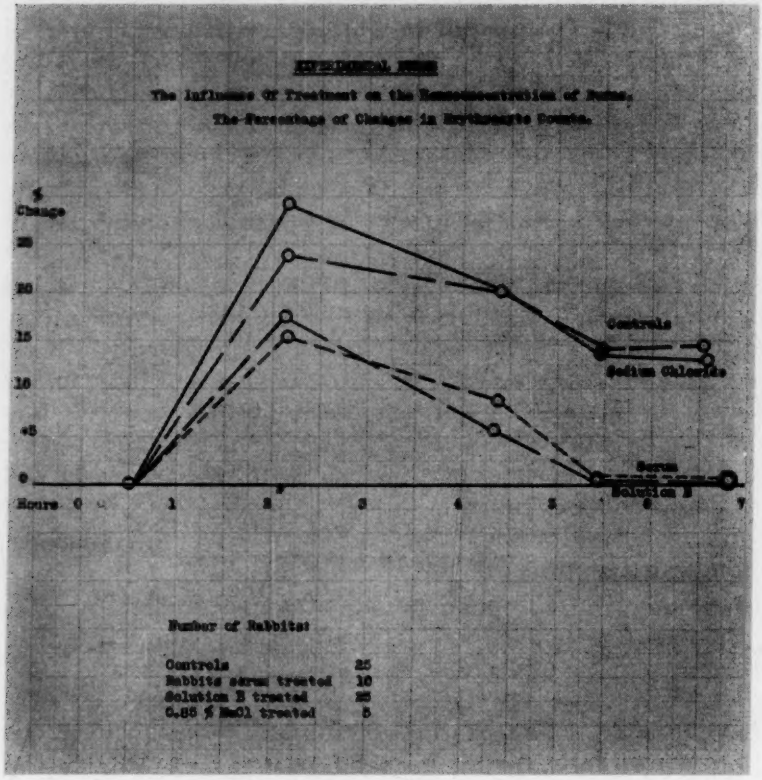


FIG. 2.

Another important discovery which may replace the serum banks and dried human serum now being collected for the treatment of shock and hemorrhage is a solution which we call solution B. This in animals acts as a perfect substitute for blood serum in the treatment of burns. We have made a considerable study of burns over more than a year and in-

volving about a score of our people. During this study, we found that solution B was equal in value to blood serum given in the same amounts and same frequency in restoring the quality of the blood in burned animals.

It is non-toxic, non-hemolytic and produces no changes in the organs and it does not produce any changes in the liver metabolism. It can be carried as a powder and diluted with water as required. It is easily made and it produces in animals a restoration of the quality of the blood commensurable with blood serum. If it fulfils the promise it should be of great value.

In our study of burns certain other things were shown. Tannic acid, a common treatment of burns, is dangerous. This was shown first by Welch of New Haven and we have amply confirmed and extended his observations.

We studied burns with the aid of the cyclotron by producing radio-active phosphorus and found that this was attracted to the burned area as well as certain other elements. There are many other phases coming out of this study of burns which are too long to detail here.

Many other phases of our work are concerned with war problems but these must remain untold at present.

So our laboratory carries on and is very active. We are about to issue our bound volume Number 6 of our work which comprises 27 papers and 24 NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION in the JOURNAL OF THE FRANKLIN INSTITUTE. These NOTES and their place of publication are of great value to us in these days of lessened publication by the scientific Journals. They appear in the JOURNAL OF THE FRANKLIN INSTITUTE every month.

**Yardstick Offered for Measuring Value of Industrial Research.**—Strangely enough, the most scientific of all pursuits, industrial research, baffles scientists and industry when it comes to valuing it in dollars. So declared EUGENE AYRES, Staff Chemist of the Gulf Research and Development Company of Pittsburgh, in a paper presented at the Division of Industrial and Engineering Chemistry of the American Chemical Society. However, from the voluminous experience of research laboratories in many industries, according to Mr. Ayres, certain criteria are beginning to emerge which can be measured, and he offered a yardstick to be used for such purposes. Mr. Ayres pointed out that while there are cases of spectacular returns from very little research, and others where research has continued year after year without tangible return, cost relationships have been pretty definitely determined by economic studies. He presented a tabulation in which units of cost were shown in three generalized cases, based on averages, for the five different stages of research. It is as follows:

| | Units of Cost. | | |
|---|---|---|---|
| 1. Laboratory Research | 1 | 1 | 0 |
| 2. Patent Prosecution | 1 | 1 | 0 |
| 3. Pilot Plant Research | 5 | 0 | 0 |
| 4. Excess Design and Construction | 5 | 15 | 20 |
| 5. Getting Process in Operation | 25 | 50 | 65 |
| Total Development Cost | 37 | 67 | 85 |

"The first column expresses the relationship between cost elements which perhaps may be regarded as ideal and, therefore, as showing a minimum total for the average research project," stated Mr. Ayres. "Laboratory research and patent prosecution are indicated as of the same order of magnitude. 'Pilot plant' expense is five times as high as 'laboratory' expense. 'Excess Design and Construction' is made equal to 'pilot plant' expense because experience indicates that this is about as low as competent research can bring it and 'Getting Process in Operation' is five times as great for the same reason. The most significant comparison is between the columns. Without any research, the 'Total Development Cost' is about 2.3 times the total cost with adequate research. 'Laboratory research' without 'pilot plant' work figures 1.8 times the ideal."

R. H. O.

# ELECTRIC PROPAGATION ON LONG LINES TERMINATED BY LUMPED NETWORKS.

BY

MICHEL G. MALTI, Ph.D.,
Associate Professor of Electrical Engineering, Cornell University,

AND

MICHAEL GOLOMB, Ph.D.,
Research Associate, Cornell University.

## PART I. LINE INITIALLY AT REST.

1. *Introduction:* Electric propagation on long lines has been studied by several authors including Pupin,[1*] Heaviside,[2] Poincaré,[3] Wagner,[4,5] Carson [6] and Doetsch.[7] Most of their work is based on one or more of the following assumptions:

*a*. The applied emf is sinusoidal and only the steady state response is investigated.
*b*. The line has neither resistance nor leakance (dissipationless).
*c*. The line is distortionless.
*d*. The line is open, shorted, or reflectionless.
*e*. The line is initially at rest.

Four authors, to our knowledge, have deviated from assumptions *a* to *d*, but maintained assumption *e*. These are F. Pollaczek,[8] L. Casper,[9] S. Hayashi,[10] and W. V. Koppenfels.[11] While maintaining assumption *e*, these authors have obtained expressions for the voltage and current which are generally known as "Travelling Wave Expansions." Each term in the expansion is given either in the form of a line integral which may be reduced to a sum of residues and real integrals,[8,10] or in the form of an infinite series of Bessel Functions [9,10,11] whose coefficients are not expressed by a general formula but must be computed by complicated manipulation of power series. In either case the numerical work of evaluating the solution is not only tedious and long but also requires the cooperation of skilled mathematicians.

* Note: Numbers refer to Bibliography.

In this paper we have attempted to obtain solutions which:

*a*. Are not restricted by any of the assumptions *a* to *e*.
*b*. Are readily evaluated numerically by persons not possessing any mathematical skill.

The solution is obtained by the well known Laplace transformation and its inversion. Its use enables one to transform the partial differential equation and its initial and boundary conditions into an ordinary differential equation of space with boundary conditions. This latter is then solved and, by inverting the transformation, an expression is obtained for the unknown function of two variables.

The particular contribution in this paper, which renders the solution well adapted to easy numerical computations, consists in expanding the transformed solution in powers of the distortion parameter and inverting this series termwise. Thus the coefficients of the resulting series are repeated convolution integrals of time functions. These functions can be either evaluated or actually measured by observations not involving the line but only the terminals. Moreover the convolution integrals lend themselves easily to numerical evaluation by mechanical devices.

In the first part of the paper we assume that the line is initially at rest. The initially excited line is dealt with in Part II.

For purposes of easy reference equations representing final results are marked by an asterisk.

2. *Differential Equations:* The differential equations of a transmission line are:

$$\left.\begin{aligned} -\frac{\partial e}{\partial x} &= Ri + L\frac{\partial i}{\partial t} && (a)\\ -\frac{\partial i}{\partial x} &= Ge + C\frac{\partial e}{\partial t}, && (b) \end{aligned}\right\} (2.1)$$

where $e = e(x, t)$, $i = i(x, t)$ are the voltage and current at the time $t$ measured from the instant $t = 0$, and at the point $x$ measured from the sending end of the line. $R$, $G$, $L$, $C$ are respectively the resistance, conductance, inductance and capacitance of the line per unit length.

Using the parameters of the line we shall define certain quantities which occur frequently in this paper:

$$\left.\begin{aligned} v &= 1/\sqrt{LC} \text{ velocity of propagation,} && (a)\\ r &= \sqrt{L/C} \text{ initial impedance of the line,} && (b)\\ \rho &= \tfrac{1}{2}(R/L + G/C) \text{ attenuation constant,} && (c)\\ \sigma &= \tfrac{1}{2}(R/L - G/C) \text{ distortion constant,} && (d)\\ \Lambda(u) &= \sqrt{(Lu+R)(Cu+G)} \text{ propagation function,} && (e)\\ Z(u) &= \sqrt{(Lu+R)/(Cu+G)} \text{ impedance function,} && (f) \end{aligned}\right\} (2.2)$$

We shall further assume for the purpose of fixing the branch of the propagation and impedance functions that:

$$\lim_{u\to\infty} \Lambda(u)/u = +\sqrt{LC}, \qquad \lim_{u\to\infty} Z(u) = +\sqrt{L/C}. \quad (2.3)$$

Eliminating either $e$ or $i$ from (2.1) through differentiation we obtain if $LC \neq 0$:

$$\left.\begin{aligned} \frac{\partial^2 e}{\partial t^2} - v^2\frac{\partial^2 e}{\partial x^2} + 2\sigma\frac{\partial e}{\partial t} + (RG/LC)e &= 0, && (a)\\ \frac{\partial^2 i}{\partial t^2} - v^2\frac{\partial^2 i}{\partial x^2} + 2\sigma\frac{\partial i}{\partial t} + (RG/LC)i &= 0. && (b) \end{aligned}\right\} (2.4)$$

If $LC = 0$ then Eqs. (2.4) do not contain the second derivative in $t$ and lead to a type of solution which is not discussed in this paper. Since zero capacitance is physically impossible the product $LC = 0$ means that $L = 0$. This is the case of the non-inductive cable.

3. *Initial Conditions:* In addition to the system of equations (2.1) or (2.4) it is necessary to supply information regarding the condition of the line at a certain instant of time. Since we assume that the line is initially at rest we have the following initial conditions:

$$\left.\begin{aligned} e(x, o) &= 0 \text{ for all } x, && (a)\\ i(x, o) &= 0 \text{ for all } x. && (b) \end{aligned}\right\} (3.1)^*$$

* Note: Eqs. (3.1) and similar equations in this paper should be interpreted as:

$$\lim_{t\to 0+} e(x, t) = 0, \qquad \lim_{t\to 0+} i(x, t) = 0.$$

From these conditions we may conclude that:

$$\left.\begin{aligned} &\frac{\partial^k e}{\partial t^k}(x, o) = 0 \text{ for all } x, \quad k = 0, 1, 2 \cdots, \quad (a) \\ &\frac{\partial^k i}{\partial t^k}(x, o) = 0 \text{ for all } x, \quad k = 0, 1, 2 \cdots. \quad (b) \end{aligned}\right\} \quad (3.2)$$

To prove this substitute (3.1) in (2.1) and conclude that (3.2) holds for $k = i$. Next differentiate (2.4), $(m - 2)$ times with respect to $t$ and obtain:

$$\left.\begin{aligned} &\frac{\partial^m e}{\partial t^m} - v^2 \frac{\partial^m e}{\partial t^{m-2} \partial x^2} + 2\sigma \frac{\partial^{m-1} e}{\partial t^{m-1}} + \frac{RG}{LC} \frac{\partial^{m-2} e}{\partial t^{m-2}} = 0 \text{ for all } x, \quad (a) \\ &\frac{\partial^m i}{\partial t^m} - v^2 \frac{\partial^m i}{\partial t^{m-2} \partial x^2} + 2\sigma \frac{\partial^{m-1} i}{\partial t^{m-1}} + \frac{RG}{LC} \frac{\partial^{m-2} i}{\partial t^{m-2}} = 0 \text{ for all } x. \quad (b) \end{aligned}\right\} \quad (3.3)$$

Eqs. (3.3) state that if (3.2) is true for $k = m - 2$ and $k = m - 1$ then it is also true for $k = m$. But, by assumption, (3.2) is true for $k = 0$, and we have shown it to be true for $k = 1$; hence it is true for $k = 2, 3, 4 \cdots$.

4. *Boundary Conditions: a. Sending End:* The line (See Fig. 1) is terminated at the sending end ($x = 0$) by a four

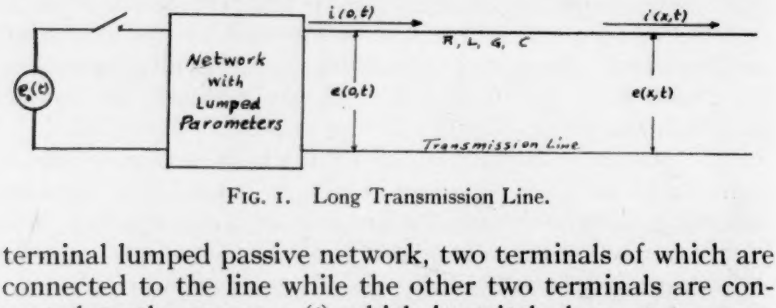


FIG. 1. Long Transmission Line.

terminal lumped passive network, two terminals of which are connected to the line while the other two terminals are connected to the source $e_0(t)$ which is switched on at $t = 0$ so that $e_0(t) = 0$ for $t < 0$.

While it is here assumed that the network is passive, such assumption is not necessary. Indeed any active network can be treated in the manner described hereafter by assuming all except one of the sources to be shorted. Solutions so obtained are then added to give the total solution.

The boundary condition for the sending end is a differential equation of the form:

$$D_1 e(o, t) + D_2 i(o, t) = D_0 e_0(t), \tag{4.1}$$

where $D_c$ are differential operators of the form:

$$D_c = \sum_{k=0}^{n_c} a_{ck} \frac{\partial^k}{\partial t^k}, \quad c = 0, 1, 2, \tag{4.2}$$

where $a_{ck}$ are functions of the parameters of the network and are always positive and where $e_0(t)$ is a given function having $n_0$ derivatives for $t < 0$ and $\lim_{t \to 0+} e_0^{(k)}(t)$ exists for $k = 0, 1, 2, \cdots (n - 1)$.

The differential Eq. (4.1) is not sufficient by itself to describe completely the conditions at the sending end. It must be accompanied by initial conditions giving the charges and currents in the passive network at $t = 0$. Since we showed, as a consequence of assumption (3.1), that the voltage and current of the line at $t = 0$ do satisfy Eqs. (3.2) it becomes necessary that restrictions be placed on the initial charges and currents of the network so that $e(x, t)$ and $i(x, t)$ shall be continuous and have all the derivatives occurring in (2.1) and (4.1) for $x = 0$, $t = 0$.

We shall show later that as $t \to 0$ the line behaves as though it were an ohmic resistance $r = \sqrt{L/C}$. Let the line be replaced by a resistance $r = \sqrt{L/C}$ and let $e_r(t)$ be the drop across this resistance $r$ and $i_r(t)$ be the current in $r$. In order that $e(x, t)$ and $i(x, t)$ shall have all the derivatives occurring in the differential Eqs. (2.1) and the initial and boundary conditions (3.2), (4.1), we shall assume that $e_r$ and $i_r$ are differentiable and that $e_r$ has $n_1$ derivatives and $i_r$ has $n_2$ derivatives for $t > 0$ and that:

$$\left.\begin{aligned} &\lim_{t \to 0} d^k e_r/dt^k = 0 \quad k = 0, 1, 2 \cdots (n_1 - 1), \quad (a) \\ &\lim_{t \to 0} d^k i_r/dt^k = 0 \quad k = 0, 1, 2 \cdots (n_2 - 1), \quad (b) \end{aligned}\right\} \tag{4.3}$$

*b. Receiving End:* The boundary condition at the receiving end shall be considered under two cases: ($b_1$) the semi-infinite

line whose boundary conditions are simply

$$\left.\begin{aligned} &\lim_{x\to\infty} e(x,t) = 0 \text{ for all } t, && (a)\\ &\lim_{x\to\infty} i(x,t) = 0 \text{ for all } t, && (b)\end{aligned}\right\} \quad (4.4)$$

($b_2$) the finite line of length $s$ which is terminated by a lumped passive network with no initial charges or currents. The boundary condition in this case is:

$$D_{1s}e(s,t) - D_{2s}i(s,t) = 0, \quad (4.5)$$

where $D_{cs}$ are differential operators of the form

$$D_{cs} = \sum_{k=0}^{n_c} a_{cks}\frac{\partial_k}{\partial t^k} \quad c = 1, 2. \quad (4.6)$$

$a_{cks}$ are functions of the parameters of the network at the receiving end and are always positive.

Observe that both boundary conditions (4.4) and (4.5) are consistent with the initial condition (3.1).

5. *Laplace Transformation and its Inversion:* In the solution of our problem we shall make use of the Laplace transformation and its inversion. A function $f(t)$ which satisfies certain conditions (See 12, p. 13) undergoes a Laplace transformation when it is multiplied by $\epsilon^{-ut}$ and integrated from zero to infinity. The resulting expression is known as Laplace transform and is denoted thus:

$$F(u) = \int_0^\infty f(t)\epsilon^{-ut}dt = L[f(t); u]. \quad (5.1)$$

It is sometimes convenient to omit the argument $u$ in the last term of (5.1) and write the transform thus:

$$F(u) = L[f(t)]. \quad (5.2)$$

The function $f(t)$ must satisfy certain conditions in order that the Laplace transformation may be applied to it. Such conditions are given in reference 12 p. 13, and functions which satisfy them are hereafter referred to as $L$-functions.

If $F(u)$ in (5.1) is known then (5.1) may be interpreted as an integral equation for the unknown function $f(t)$. It can

be shown (12, p. 33) that if: $F(u) = L[f_1(t)] = L[f_2(t)]$ then $f_1(t) = f_2(t)$ at every point $(t)$ where $f_1(t)$ or $f_2(t)$ is continuous. Hence the inverse of a Laplace transform is unique except at the points of discontinuity. We shall denote the unique inversion thus:

$$f(t) = L^{-1}[F(u); t] \quad \text{or} \quad f(t) = L^{-1}[F(u)]. \tag{5.3}$$

One of the best known solutions of the integral equation (5.1) is:

$$f(t) = L^{-1}[F(u)] = \frac{1}{2\pi j}\int_{b-j\infty}^{b+j\infty} F(u)\epsilon^{ut}du. \tag{5.4}$$

Other forms of expressing $f(t)$ through $F(u)$ are known.[12] In order to clarify our notations and terminology capital letters shall be used for the transforms and the corresponding small letters for the inverses. We shall also state that $f(t)$ is "transformed" into $F(u)$ and $F(u)$ is "inverted" into $f(t)$.

We shall make use of certain transformation rules and theorems regarding Laplace transforms. These are given in an appendix to this paper.

6. *Transformed Problem:* Applying (A.2), Appendix, to the differential equations (2.1) and observing the initial conditions (3.1) we have

$$\left.\begin{aligned} -\frac{\partial E}{\partial x} &= (R + Lu)I, \qquad (a)\\ -\frac{\partial I}{\partial x} &= (G + Cu)E, \qquad (b)\end{aligned}\right\} \tag{6.1}$$

where $E$ and $I$ are the Laplace transforms of $e$ and $i$ respectively. Observe that (6.1) are ordinary differential equations in $E$ and $I$ with $x$ as the independent variable and $u$ a parameter.

Again applying (A.2), Appendix, to the sending end boundary condition (4.1) and taking into account relations (3.2) we have:

$$D_1(u)E(0, u) + D_2(u)I(0, u) = D_0(u)E_0(u) - P_0(u), \tag{6.2}$$

where $E_0(u)$ is the transform of $e_0(t)$, $D_0(u)$, $D_1(u)$, $D_2(u)$ are the expressions (4.2) with $\partial^k/\partial t^k$ replaced by $u^k$ and $P_0(u)$ is

a polynomial defined as follows:

$$\left.\begin{aligned} P_0(u) &= \sum_{k=1}^{n_0} A_{k1} + uA_{k2} + u^2A_{k3} + \cdots u^{k-1}A_{kk}, \quad (a)\\ A_{ks} &= a_{ok}e_0{}^{(k-s)}(0). \quad (b)\end{aligned}\right\} \quad (6.3)$$

In the same way the receiving end boundary conditions (4.4) and (4.5) are transformed, giving:

*For the infinite line:*

$$\lim_{x\to\infty} E(x, u) = 0, \qquad \lim_{x\to\infty} I(x, u) = 0 \text{ for all } t. \quad (6.4)$$

*For the finite line:*

$$D_{1s}(u)E(s, u) - D_{2s}(u)I(s, u) = 0. \quad (6.5)$$

7. *Transformed Solution of the Infinite Line:* The solution of the system of ordinary differential equations (6.1) with their boundary conditions (6.2) and (6.4) is:

$$\left.\begin{aligned} E(x, u) &= Z(u)\frac{E_0(u)D_0(u) - P_0(u)}{Z(u)D_1(u) + D_2(u)}\epsilon^{-x\Delta(u)}, \quad (a)\\ I(x, u) &= \frac{E_0(u)D_0(u) - P_0(u)}{Z(u)D_1(u) + D_2(u)}\epsilon^{-x\Delta(u)}. \quad (b)\end{aligned}\right\} \quad (7.1)$$

Let

$$I_0 = I(0, u) = \frac{E_0(u)D_0(u) - P_0(u)}{Z(u)D_1(u) + D_2(u)}. \quad (7.2)$$

Omit, for brevity, the arguments $x$, $u$, substitute (7.2) in (7.1) and obtain:

$$\left.\begin{aligned} E &= ZI_0\epsilon^{-x\Delta}, \quad (a)\\ I &= I_0\epsilon^{-x\Delta}. \quad (b)\end{aligned}\right\} \quad (7.3)$$

It is necessary, for later purposes, to express $I_0$ in an expanded form. To do this write (7.2) as follows, using the notations introduced in (2.2):

$$I_0 = \frac{E_0D_0 - P_0}{rD_1 + D_2}\frac{1}{1 - \dfrac{1 - Z/r}{1 + D_2/rD_1}}. \quad (7.4)$$

Since $D_1$ and $D_2$ are polynomials with non-negative coefficients, the real part of the term $D_2/rD_1$ is positive for all $u$ with sufficiently large $Ru$. Again $\lim_{u\to\infty} Z(u) = \sqrt{L/C} = r$. Therefore, for sufficiently large $Ru$, there exists a constant $k < 1$ such that

$$\left|\frac{1 - Z/r}{1 + D_2/rD_1}\right| \leqq k < 1. \tag{7.5}$$

Thus the second factor in the product (7.4) can be expanded as a geometric series which is absolutely and uniformly convergent in the half plane $Ru \geqq \alpha$, giving:

$$I_0 = \frac{E_0D_0 - P_0}{rD_1 + D_2}\sum_{n=0}^{\infty}(1 - Z/r)^n(1 + D_2/rD_1)^{-n}. \tag{7.6}$$

8. *Inversion of Eqs.* (7.3): In inverting (7.3) we recognize the existence of the products $I_0(Z\epsilon^{-x\Delta})$ and $I_0\epsilon^{-x\Delta}$. The inversion of these products suggests the use of the convolution theorem. It will be shown that the inverse $i_0(t)$ of $I_0$ exists and $i_0(t)$ will be evaluated in the succeeding two paragraphs. But the inverses of $Z\epsilon^{-x\Delta}$ and $\epsilon^{-x\Delta}$ do not exist. On the other hand if we subtract an exponential term from each of these functions then the inverse of the remainder function does exist. We shall therefore find the inverses of these remainder functions, apply the convolution theorem and split the resulting expression into terms which represent the transforms of (7.3) and terms which arose from the subtracted function. The former represent the inversion of (7.3) while the latter are cancelled.

Substitute (2.2) in (7.3) and obtain

$$\left.\begin{aligned} E &= I_0r\left\{\sqrt{\frac{u + R/L}{u + G/C}}\right. \\ &\qquad \left.\times \exp\left[-\frac{x}{v}\sqrt{\left(u + \frac{R}{L}\right)\left(u + \frac{G}{C}\right)}\right]\right\}, \quad (a) \\ I &= I_0\left\{\exp\left[-\frac{x}{v}\sqrt{\left(u + \frac{R}{L}\right)\left(u + \frac{G}{C}\right)}\right]\right\}. \quad (b) \end{aligned}\right\} \tag{8.1}$$

Now consider the factors in braces of Eqs. (8.1) and sub-

tract from each of them $\exp\left[-\frac{x}{v}(u+\rho)\right]$ and obtain:

$$\left.\begin{aligned} A(u) &= \sqrt{\frac{u+R/L}{u+G/C}}\exp\left[-\frac{x}{v}\sqrt{\left(u+\frac{R}{L}\right)\left(u+\frac{G}{C}\right)}\right] \\ &\qquad - \exp\left[-\frac{x}{v}(u+\rho)\right], \quad (a) \\ B(u) &= \exp\left[-\frac{x}{v}\sqrt{\left(u+\frac{R}{L}\right)\left(u+\frac{G}{C}\right)}\right] \\ &\qquad - \exp\left[-\frac{x}{v}(u+\rho)\right]. \quad (b) \end{aligned}\right\} (8.2)$$

The inverses of these functions are well known. They may, for example, be found in Reference 14 Formula 863.1 and 864.1. They are:

$$\left.\begin{aligned} a(t) &= L^{-1}[A(u)] = \mathbf{1}\left(t-\frac{x}{v}\right) S_e(t, x/v); \quad x \geqq 0, \quad (a) \\ b(t) &= L^{-1}[B(u)] = \mathbf{1}\left(t-\frac{x}{v}\right) S_i(t, x/v); \quad x \geqq 0, \quad (b) \end{aligned}\right\} (8.3)$$

where $\mathbf{1}$ is Heaviside's unit function, $S_e$ and $S_i$ are the "distortion kernels" defined by:

$$\left.\begin{aligned} S_e(t, y) &= \sigma\epsilon^{-\rho t}\left[\frac{t}{\sqrt{t^2-y^2}}\,\mathfrak{I}_1(\sigma\sqrt{t^2-y^2})\right. \\ &\qquad \left. + \mathfrak{I}_0(\sigma\sqrt{t^2-y^2})\right], \quad (a) \\ S_i(t, y) &= \sigma y\epsilon^{-\rho t}\frac{\mathfrak{I}_1(\sigma\sqrt{t^2-y^2})}{\sqrt{t^2-y^2}}. \quad (b) \end{aligned}\right\} (8.4)$$

$\mathfrak{I}_0$ and $\mathfrak{I}_1$ are the Bessel functions of imaginary argument of order 0 and 1 respectively.

For later use we remark that the expansions of $\mathfrak{I}_0$ and $\mathfrak{I}_1$ in powers of $\sigma$ yield:

$$\left.\begin{aligned} S_e(t, y) &= \sigma\epsilon^{-\rho t} + ((\sigma^2)), \quad (a) \\ S_i(t, y) &= 0 + ((\sigma^2)). \quad (b) \end{aligned}\right\} (8.5)$$

Applying the convolution theorem to the products $I_0 rA(u)$ and $I_0 B(u)$ we obtain:

$$\left.\begin{aligned} L^{-1}[I_0 rA] &= r[i_0 * a]_t \\ &= \mathbf{1}(t - x/v) r \int_{x/v}^{t} S_e(\tau, x/v) i_0(t - \tau) d\tau, \quad (a) \\ L^{-1}[I_0 B] &= [i_0 * b]_t \\ &= \mathbf{1}(t - x/v) \int_{x/v}^{t} S_i(\tau, x/v) i_0(t - \tau) d\tau. \quad (b) \end{aligned}\right\} (8.6)$$

But by Heaviside's shifting:

$$L^{-1}\left[ I_0 \exp\left\{ -\frac{x}{v}(u + \rho) \right\} \right] = \mathbf{1}(t - x/v) \epsilon^{-\rho x/v} i_0(t - x/v). \quad (8.7)$$

Adding (8.7) to (8.6) we have the inverses of $E$ and $I$ of (8.1) respectively or:

$$\left.\begin{aligned} e(x, t) &= L^{-1}[E] = \mathbf{1}(t - x/v) r \\ &\quad \times \left[ \epsilon^{-\rho x/v} i_0(t - x/v) + \int_{x/v}^{t} S_e(\tau, x/v) i_0(t - \tau) d\tau \right] \quad (a) \\ i(x, t) &= L^{-1}[I] = \mathbf{1}(t - x/v) \\ &\quad \times \left[ \epsilon^{-\rho x/v} i_0(t - x/v) + \int_{x/v}^{t} S_i(\tau, x/v) i_0(t - \tau) d\tau \right]. \quad (b) \end{aligned}\right\} (8.8)^*$$

It is significant to show that $e(o, t)$ and $i(o, t)$ behave, as $t \to 0$, like the functions $e_r$ and $i_r$ which were defined in paragraph 4*a*. More exactly:

$$\left.\begin{aligned} \operatorname*{Lim}_{t \to 0+} \frac{d^k}{dt^k} e(o, t) &= \operatorname*{Lim}_{t \to 0+} \frac{d^k}{dt^k} e_r(t) = 0, \\ &\qquad k = 0, 1, 2 \cdots (n_1 - 1), \quad (a) \\ \operatorname*{Lim}_{t \to 0+} \frac{d^k}{dt^k} i(o, t) &= \operatorname*{Lim}_{t \to +0} \frac{d^k}{dt^k} i_r(t) = 0, \\ &\qquad k = 0, 1, 2 \cdots (n_2 - 1). \quad (b) \end{aligned}\right\} (8.9)$$

To prove this statement we observe that the transforms $E_r$ and $I_r$ of $e_r$ and $i_r$ are obtained by substituting $x = 0$ and

$Z = r$ in (7.1) or:

$$E_r = L[e_r] = r\frac{E_0D_0 - P_0}{rD_1 + D_2}, \qquad I_r = L[i_r] = \frac{E_0D_0 - P_0}{rD_1 + D_2}. \tag{8.10}$$

On the other hand, substituting $x = 0$ in (7.1), we have:

$$E(o, u) = Z\frac{E_0D_0 - P_0}{ZD_1 + D_2}, \qquad I(o, u) = \frac{E_0D_0 - P_0}{ZD_1 + D_2}. \tag{8.11}$$

From (8.10) and (8.11) we have:

$$\left.\begin{aligned} E(o, u) &= E_r(1 - \Phi_e), \quad (a)\\ I(o, u) &= I_r(1 + \Phi_i), \quad (b)\end{aligned}\right\} \tag{8.12}$$

where

$$\Phi_e = \frac{(1 - Z/r)D_2}{ZD_1 + D_2}, \qquad \Phi_i = \frac{(1 - Z/r)D_1}{ZD_1 + D_2}. \tag{8.13}$$

Since $\lim_{u\to\infty}(1 - Z/r) = 0$ the hypotheses of theorem *e*, Appendix, are seen to be satisfied, and therefore the conclusion (8.9) is true.

A further conclusion from theorem *e* is that $I_0(u) = I(o, u)$ is the Laplace transform of a function $i_0(t)$ which is differentiable and has $n_2$ derivatives.

9. *Eqs. (8.8) are Solutions:* The functions (8.8) are not only the inverses of (7.3) but are actually the solutions of our problem. To demonstrate this fact it is necessary to show that the functions (8.8) satisfy the differential equations (2.1) and also the boundary and initial conditions imposed.

To show that (8.8) satisfy the differential equations (2.1) observe that $i_0(t)$ is differentiable. Hence both $e(x, t)$ and $i(x, t)$ have derivatives with respect to $x$ and $t$. That these derivatives satisfy (2.1) may be readily seen by actually performing the differentiation and substituting. That (8.8) satisfy the initial conditions (3.1) is deduced as follows:

(*a*) if $t = 0$ and $x > 0$ then by definition of the unit function $\mathbf{1}(t - x/v) = 0$. Hence $e(x, o) = i(x, o) = 0$.

(*b*) if $t = 0$ and $x = 0$ then $i_0(t - x/v) = 0$ by (8.9). Hence $e(0, 0) = i(0, 0) = 0$.

That (8.8) satisfy the boundary conditions (4.4) follows from the fact that, by definition, $\mathbf{1}(t - x/v) = 0$ at $x = \infty$ for all $t$.

To show that (8.8) satisfy the boundary condition (4.1) observe that their transforms (7.1) satisfy the transformed boundary condition (6.2). Moreover $e$ and $i$ satisfy the relations (8.9). Hence by (A2) of Appendix, $e(x, t)$ and $i(x, t)$ satisfy the boundary condition (4.1).

For purposes of future reference we shall designate the functions $e(x, t)$ and $i(x, t)$ for a semi-infinite line satisfying the initial conditions (3.1) and the sender conditions (4.4) by $e_\infty$ and $i_\infty$ thus:

$$\left.\begin{aligned} e_\infty(x, t) &= \mathbf{1}(t - x/v)r \\ &\quad \times \left[ \epsilon^{-\rho x/v} i_0(t - x/v) + \int_{x/v}^{t} S_e(\tau, x/v) i_0(t - \tau) d\tau \right], \quad (a) \\ i_\infty(x, t) &= \mathbf{1}(t - x/v) \\ &\quad \times \left[ \epsilon^{-\rho x/v} i_0(t - x/v) + \int_{x/v}^{t} S_i(\tau, x/v) i_0(t - \tau) d\tau \right]. \quad (b) \end{aligned}\right\} (9.1)^*$$

10. *Inversion of $I_0$:* In the solutions (8.8) we used $i_0(t)$ as the inverse of $I_0(u)$; we did not, however, evaluate this inverse, which we now proceed to do, dividing our work into two cases according to whether the line is or is not distortionless.

*a. The distortionless line* ($\sigma = 0$): If $\sigma = 0$ then $Z = r$ and $I_0(u)$ (in Eq. 7.2) becomes $I_r$ (Eq. 8.10). Write it in the form

$$I_0(u) = I_r = \frac{D_0}{rD_1 + D_2} E_0 - \frac{P_0}{rD_1 + D_2}. \qquad (10.1)$$

Since $D_0/(rD_1 + D_2)$ and $P_0/(rD_1 + D_2)$ are rational functions their inverses are readily known (See *c2* and *d*, Appendix). Let their inverses be respectively:

$$\left.\begin{aligned} \mu(t) &= L^{-1}\left[ \frac{D_0}{rD_1 + D_2} \right], \quad (a) \\ \eta(t) &= L^{-1}\left[ \frac{P_0}{rD_1 + D_2} \right]. \quad (b) \end{aligned}\right\} (10.2)^*$$

Using these inverses and applying the convolution theorem to (10.1), we have:

$$i_0 = i_r = \mu * e_0 - \eta. \qquad (10.3)^*$$

Since (see 8.5) when $\sigma = 0$ the kernels $S_e$ and $S_i$ are zero, Eqs. (9.1) reduce in this case to:

$$\left.\begin{aligned} e_\alpha(x,t) &= \mathbf{1}(t - x/v) r i_r(t - x/v)\epsilon^{-\rho x/v}, \quad (a)\\ i_\alpha(x,t) &= \mathbf{1}(t - x/v) i_r(t - x/v)\epsilon^{-\rho x/v}, \quad (b)\end{aligned}\right\}(10.4)^*$$

where $i_r$ is given in (10.3).

*b. The Distortive Line* ($\sigma \neq 0$): We have seen that the solution (10.4), for a distortionless line, is very simple. Physically, the smaller the distortion $\sigma$ the nearer would the solution be to (10.4). The appropriate procedure would therefore be to express $i_0(t)$ in powers of $\sigma$ and consider only enough terms of the expansion to give satisfactory numerical accuracy. This is the procedure we adopt.

Write (2.2*f*) as follows:

$$\left.\begin{aligned} Z &= r\sqrt{1 + \frac{2\sigma}{u + G/C}}, \quad (a)\\ 1 - \frac{Z}{r} &= 1 - \sqrt{1 + \frac{2\sigma}{u + G/C}}. \quad (b)\end{aligned}\right\}(10.5)$$

Apply Lagrange's theorem, 15 p. 132 Eq. 2, to (10.5*b*) and obtain the following series:

$$(1 - Z/r)^n = \sum_{m=0}^{\infty} (-1)^{n+m} 2^{-m} \times \frac{n}{n+2m}\binom{n+2m}{n+m}\left(\frac{\sigma}{u + G/C}\right)^{n+m}. \quad (10.6)$$

The series (10.6) converges uniformly and absolutely in some half plane $Ru \geqq \alpha$.

Substitute (10.6) in (7.6) and obtain

$$\left.\begin{aligned} I_0 &= \frac{E_0 D_0 - P_0}{rD_1 + D_2}\sum_{n=0}^{\infty}\left[\sum_{m=0}^{\infty}\frac{(-1)^{n+m}2^{-m}n}{n+2m} \times \binom{n+2m}{n+m}\left(\frac{\sigma}{u+G/C}\right)^{n+m}\right]\left(1 + \frac{D_2}{rD_1}\right)^{-n}, \quad (a)\\ &= \sum_{k=0}^{\infty} I_{0k}\sigma^k, \quad (b)\end{aligned}\right\}(10.7)$$

where

$$\left.\begin{aligned} I_{00} &= \frac{E_0D_0 - P_0}{rD_1 + D_2} = I_r \text{ (See 8.10)}, \qquad (a)\\ I_{0k} &= (-1)^k \frac{I_r}{(u + G/C)^k} \sum_{m=0}^{k-1} 2^{-m} \frac{k - m}{k + m} \\ &\quad \times \binom{k+m}{k} \left(\frac{rD_1}{rD_1 + D_2}\right)^{k-m}, \quad k = 1, 2 \cdots. \qquad (b) \end{aligned}\right\} (10.8)$$

Since the series (10.7) converges uniformly and absolutely, it may be inverted termwise.[12.p398*] The inverse of $I_r$ is $i_r$ (See Eq. 10.3). In order to invert $I_{0k}$ we observe that $rD_1/(rD_1 + D_2)$ is a rational function whose inverse is known. Let it be

$$\nu(t) = L^{-1}\left[\frac{rD_1}{rD_1 + D_2}\right]. \qquad (10.9)^*$$

Again the inverse of $(u + G/C)^{-k}$ is known:

$$\frac{t^{k-1}\epsilon^{-tG/C}}{(k-1)!} = L^{-1}\left[\frac{1}{(u + G/C)^k}\right], \quad k = 1, 2 \cdots. \qquad (10.10)$$

Using (10.9), (10.10), the convolution theorem, and inverting (10.8) termwise we have:

$$i_{0k}(t) = L^{-1}[I_{0k}] = \frac{(-1)^k}{(k-1)!}\left[ i_r * (t^{k-1}\epsilon^{-tG/C}) * \sum_{m=0}^{k-1} 2^{-m} \times \frac{k-m}{k+m}\binom{k+m}{k} \nu^{*(k-m)} \right]_t. \qquad (10.11)^*$$

Having inverted each of the terms of the series (10.7) we finally have as the inverse of $I_0$:

$$i_0 = L^{-1}[I_0] = \sum_{k=0}^{\infty} \sigma^k i_{0k}. \qquad (10.12)^*$$

Eq. (10.12) together with (10.11), (10.9), (10.3) and (8.8) gives a complete solution of the problem of the distortive line. It is seen that the solution involves the use of several convolution integrals whose evaluation involves many numerical computations. Several machines have been invented to lighten this work. In our opinion the best suited machine

* Note: 12.p398 means reference 12, page 398.

for the evaluation of convolution integrals is the cinema integraph [16] which could possibly be improved to adapt it to the rapid evaluation of repeated convolution integrals such as occur here.

Substituting (10.11) in (10.12) and the result in (8.8) and expanding $I_0$ and $I_1$ of (8.4) in powers of $\sigma$ and substituting this expansion in (8.8), we have expressions of $e$ and $i$ as power series in $\sigma$. The first and second terms of these series are (See (8.5) for the expansion of $S_e$ and $S_i$):

$$\left.\begin{aligned} e_\alpha(x,t) &= \mathbf{1}(t-x/v)r\epsilon^{-\rho x/v} \\ &\quad \times [i_r + \sigma i_{01} + \sigma i_r * \epsilon^{-\rho t}]_{t-x/v} + ((\sigma^2)), \quad (a) \\ i_\alpha(x,t) &= \mathbf{1}(t-x/v)\epsilon^{-\rho x/v}[i_r + \sigma i_{01}]_{t-x/v} + ((\sigma^2)), \quad (b) \end{aligned}\right\} \quad (10.13)^*$$

where (See 10.11)

$$i_{01} = -[i_r * \epsilon^{-tG/C} * \nu]. \quad (10.14)^*$$

$i_r$ and $\nu$ are respectively given by (10.3) and (10.9).

The number of terms of the expansion which must be used to obtain a given degree of accuracy depends on the characteristics of the line and the boundary conditions. It is thus impossible to make a general statement on this matter. However, in a given case, the number of terms can be readily ascertained by using any of the well known formulas [15 p 95] for the remainder of an absolutely and uniformly convergent power series.

11. *Transformed Solution of the Finite Line:* The solution of the system of ordinary differential Eqs. (6.1) with their boundary conditions (6.2) and (6.5) is:

$$\left.\begin{aligned} E(x,u) &= Z(u)I_0 \frac{\epsilon^{-x\Delta} - K_s\epsilon^{-(2s-x)\Delta}}{1 - K_0K_s\epsilon^{-2s\Delta}}, \quad (a) \\ I(x,u) &= I_0 \frac{\epsilon^{-x\Delta} + K_s\epsilon^{-(2s-x)\Delta}}{1 - K_0K_s\epsilon^{-2s\Delta}}, \quad (b) \end{aligned}\right\} \quad (11.1)$$

where

$$K_0 = \frac{ZD_1 - D_2}{ZD_1 + D_2}, \qquad K_s = \frac{ZD_{1s} - D_{2s}}{ZD_{1s} + D_{2s}}. \quad (11.2)$$

In order to find the inverses of $E$ and $I$ in (11.1) we shall express $1/(1 - K_0K_s\epsilon^{-2s\Delta})$ as a geometric series. This can be done because, for sufficiently large $Ru$, there exists a constant

$k < 1$ such that:

$$|K_0K_s\epsilon^{-2s\Delta}| \leqq k < 1. \tag{11.3}$$

Performing this expansion and substituting it in (11.1) we have:

$$\left.\begin{aligned} E &= ZI_{0\epsilon}{}^{-x\Delta} + ZI_0 \sum_{n=1}^{\infty} [(K_0K_s)^n\epsilon^{-(2ns+x)\Delta} \\ &\qquad - K_s(K_0K_s)^{n-1}\epsilon^{-(2ns-x)\Delta}], \quad (a) \\ I &= I_{0\epsilon}{}^{-x\Delta} + I_0 \sum_{n=1}^{\infty} [(K_0K_s)^n\epsilon^{-(2ns+x)\Delta} \\ &\qquad + K_s(K_0K_s)^{n-1}\epsilon^{-(2ns-x)\Delta}]. \quad (b) \end{aligned}\right\} \tag{11.4}$$

The series in (11.4) converge absolutely and uniformly with respect to $u$, for sufficiently large $Ru$.

12. *Inversion of Eqs. (11.4)*: In Eq. (11.4) all the terms have been inverted except $(K_0K_s)^n$ and $K_s(K_0K_s)^{n-1}$. Thus $ZI_{0\epsilon}{}^{-x\Delta}$ and $I_{0\epsilon}{}^{-x\Delta}$ are identical with (7.3) and their inversion is given by (9.1). Again, since $ZI_{0\epsilon}{}^{-(2ns\pm x)\Delta}$ and $I_{0\epsilon}{}^{-(2ns\pm x)\Delta}$ are expressions similar to (7.3), with $x$ replaced by $(2ns \pm x)$, their inversion is given by (9.1) with $x$ replaced by $(2ns \pm x)$. In the succeeding paragraph we shall give the inversion of $K_0$ and $K_s$. For the present let these inversions be:

$$k_0 = L^{-1}[K_0], \qquad k_s = L^{-1}[K_s]. \tag{12.1}$$

Using these facts and the convolution theorem, Eqs. (11.4) are inverted as follows:

$$\left.\begin{aligned} e(x, t) &= e_\alpha(x, t) + \sum_{n=1}^{\infty} [k_0{}^{*n} * k_s{}^{*n} * e_\alpha(2ns + x, t) \\ &\qquad - k_0{}^{*(n-1)} * k_s{}^{*n} * e_\alpha(2ns - x, t)], \quad (a) \\ i(x, t) &= i_\alpha(x, t) + \sum_{n=1}^{\infty} [k_0{}^{*n} * k_s{}^{*n} * i_\alpha(2ns + x, t) \\ &\qquad + k_0{}^{*(n-1)} * k_s{}^{*n} * i_\alpha(2ns - x, t)]. \quad (b) \end{aligned}\right\} (12.2)^* \tag{12.2}$$

It must be observed that the factor $\mathbf{1}(t - x/v) = 0$ if $x/v > t$. Hence (See 9.1) $e_\alpha(2ns \pm x, t)$ and $i_\alpha(2ns \pm x, t)$ will each be zero when $(2ns \pm x)/v > t$. Thus all terms in the series (12.2) will vanish whenever $n$ is such that $(2ns \pm x)/v > t$. The series therefore are finite for any fixed $t$.

13. *Eqs. (12.2) are Solutions:* That the functions (12.2) satisfy the differential Eqs. (2.1), the initial conditions (3.1),

and the boundary condition (4.1) is shown exactly as in Art. 9. To show that they do satisfy the boundary condition (4.5) observe that their transforms (11.1) satisfy the transformed boundary condition (6.5). Moreover, because of the factor $\mathbf{1}(t - x/v)$, we have:

$$\lim_{t\to 0+} \frac{d^k}{dt^k} e(s, t) = 0, \qquad \lim_{t\to 0+} \frac{d^k}{dt^k} i(s, t) = 0, \qquad k = 0, 1 \cdots.$$

Hence by (A.2) of Appendix, $e(x, t)$ and $i(x, t)$ satisfy the boundary condition (4.5).

Let us next examine the solutions (12.2) physically. To do this consider a fixed point $x_0$ somewhere between the terminals of the line $x = 0$ and $x = s$. Now let the time $t$ vary and substitute different values of $t$, lying in certain intervals, in Eq. (12.2$a$). We have

$$\left.\begin{aligned}
&e(x_0, t) = 0 \quad \text{for} \quad t < t_1 = x_0/v, && (a)\\
&e(x_0, t) = e_\alpha \quad \text{for} \quad t_1 < t < t_2 = t_1 + (2s - x_0)/v, && (b)\\
&e(x_0, t) = e_\alpha(x_0, t) - k_s * e_\alpha(2s - x_0, t)\\
&\qquad\qquad \text{for} \quad t_2 < t < t_3 = t_2 + \frac{2s}{v}, && (c)\\
&e(x_0, t) = e_\alpha - k_s * e_\alpha(2s - x, t)\\
&\qquad\qquad + k_0 * k_s * e_\alpha(4s + x_0, t)\\
&\qquad\qquad \text{for} \quad t_3 < t < t_4 = t_3 + \frac{(2s - x_0)}{v}, && (d)
\end{aligned}\right\} \quad (13.1)$$

etc.

Similar expressions are obtained for $i(x_0, t)$.
Physically, this means:

*a*. During the interval $0 < t < t_1$ there exists no voltage at $x = x_0$.

*b*. During the interval $t_1 < t < t_2$ the only voltage at $x = x_0$ is the direct wave of voltage encountered with infinite lines.

*c*. During the interval $t_2 < t < t_3$ there exist two voltage waves at $x = x_0$, viz. the direct wave $e_\alpha$ and a wave reflected from the receiving end, which is represented by the second term to the right of (13.1*c*).

*d*. During the interval $t_3 < t < t_4$ there exist three voltage waves at $x = x_0$, viz. the direct wave $e_\infty$, a wave reflected from the receiving end, represented by the second term in (13.1*d*) and another wave reflected from the sending end, which is represented by the third term in (13.1*d*).

As time goes on other terms representing reflections from both the sending and receiving ends of the line are added to the terms of (13.1*d*). In general, the term $k_0^{*n} * k_s^{*n} * e_\infty(2ns + x, t)$ represents a wave which has undergone $n$ reflections from the receiving and $n$ reflections from the sending end while $k_0^{*(n-1)} * k_s^{*n} * e_\infty(2ns - x, t)$ represents a wave reflected $n$ times from the receiving end and $n - 1$ times from the sending end. Each reflection from the receiving end is mathematically represented by the convolution factor $k_s$ while each reflection from the sending end is represented by the convolution factor $k_0$. We call the functions $k_0$ and $k_s$ the reflection kernels for the sending and receiving end respectively, and now proceed to evaluate them.

14. *Evaluation of $k_0$ and $k_s$:* Write $K_0$, Eq. (11.2), in the following form

$$K_0 = 1 - \frac{2D_2}{ZD_1 + D_2}. \tag{14.1}$$

Observe that the denominator of (14.1) is identical with that of $I_0$ in (7.2). Hence using the expansion for $I_0$ (See Eq. 10.7) we obtain for $K_0$:

$$\left.\begin{aligned} K_0 &= \sum_{k=0}^{\infty} K_{0k}\sigma^k, && (a)\\ &\text{where}\\ K_{00} &= \frac{rD_1 - D_2}{rD_1 + D_2} = \frac{2rD_1}{rD_1 + D_2} - 1, && (b)\\ K_{0k} &= (-1)^{k-1}\left(\frac{rD_1}{rD_1 + D_2} - 1\right)(u + G/C)^{-k}\\ &\quad \times \sum_{m=0}^{k-1} 2^{-m+1}\frac{k-m}{k+m}\binom{k+m}{k}\left(\frac{rD_1}{rD_1 + D_2}\right)^{k-m}, && (c)\\ &\qquad\qquad k = 1, 2, \cdots. \end{aligned}\right\} \tag{14.2}$$

Now the inverses of each of the terms in (14.2*b*) and (*c*) have already been introduced (See 10.9, 10.10). Hence, by the convolution theorem:

$$\left.\begin{aligned} k_{00} &= L^{-1}[K_{00}] = 2\nu - \delta_0, && (a)\\ k_{0k} &= \frac{(-1)^{k-1}}{(k-1)!}\Big[ (\nu - \delta_0) * t^{k-1}\epsilon^{-tG/C} * \sum_{m=0}^{k-1} 2^{-m+1} \\ &\qquad \times \frac{k-m}{k+m}\binom{k+m}{k} \nu^{*(k-m)} \Big]_t, && (b)\\ &\qquad\qquad k = 1, 2, \cdots. \end{aligned}\right\} (14.3)^*$$

Having inverted each of the terms in (14.2*a*) we have, for the inversion of $K_0$,

$$k_0 = L^{-1}[K_0] = \sum_{k=0}^{\infty} \sigma^k k_{0k}. \tag{14.4}$$

As to the inversion of $K_s$ observe that in (11.2) both $K_0$ and $K_s$ are of the same form except that the subscripts $s$ accompany the $D$'s. Hence if we introduce the inverse

$$\nu_s(t) = L^{-1}\left[\frac{rD_{1s}}{rD_{1s} + D_{2s}}\right], \tag{14.5}$$

and proceed in a manner similar to that adopted for the inversion of $K_0$ we have

$$k_s = L^{-1}[K_s] = \sum_{k=0}^{\infty} \sigma^k k_{sk}, \tag{14.6}$$

where (See Eq. 14.3)

$$\left.\begin{aligned} k_{s0} &= 2\nu_s - \delta_0, && (a)\\ k_{sk} &= \frac{(-1)^{k-1}}{(k-1)!}\Big[ (\nu_s - \delta_0) * t^{k-1}\epsilon^{-tG/C} * \sum_{m=0}^{k-1} 2^{-m+1} \\ &\qquad \times \frac{k-m}{k+m}\binom{k+m}{k} \nu_s^{*}(k-m) \Big]_t, && (b)\\ &\qquad\qquad k = 1, 2, \cdots. \end{aligned}\right\} (14.7)^*$$

For purposes of approximate computations we give the

first two terms of $k_0$ and $k_s$ as computed from (14.3) and (14.7)

$$\left.\begin{aligned} k_0 &= (2\nu - \delta_0) + 2\sigma(\nu - \delta_0) * \nu * \epsilon^{-tG/C} + ((\sigma^2)), \quad (a) \\ k_s &= (2\nu_s - \delta_0) + 2\sigma(\nu_s - \delta_0) * \nu_s * \epsilon^{-tG/C} + ((\sigma^2)). \quad (b) \end{aligned}\right\} (14.8)^*$$

15. *Experimental Determination of* $i_r$, $\nu(t)$, $\nu_s(t)$: Reference to Eqs. (10.4), (10.13) and (12.2) and (14.8) shows that three inverses occur in all the solutions. These are:

$$\left.\begin{aligned} i_r(t) &= L^{-1}[I_r] = L^{-1}\left[\frac{E_0D_0 - P_0}{rD_1 + D_2}\right], \\ &\qquad \text{see Eqs. (8.10)}, \quad (a) \\ \nu(t) &= L^{-1}\left[\frac{rD_1}{rD_1 + D_2}\right], \text{ see Eq. (10.9)}, \quad (b) \\ \nu_s(t) &= L^{-1}\left[\frac{rD_{1s}}{rD_s + D_{2s}}\right], \text{ see Eq. (14.5)}. \quad (c) \end{aligned}\right\} (15.1)$$

These inverses involve the inversion of rational functions which depend upon the terminal conditions of the line and, in some instances, might lead to rather complicated expressions. We give, therefore, an experimental procedure which enables one to obtain oscillograms of these functions directly.

*a. Experimental Determination of* $i_r(t)$: We have shown (See Art. 8) that $i_r$ as defined by (15.1*a*) is the same as the current across the resistance $r = \sqrt{L/C}$ (See Art. 4) which replaces the transmission line. Hence to obtain $i_r$ experimentally simply replace the line by $r = \sqrt{L/C}$, impress the given emf, and take an oscillogram of the current $i_r$ in the resistance $r$.

*b. Experimental Determination of* $\nu(t)$: Referring to Fig. 2*a*:

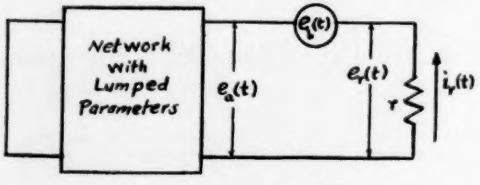


FIG. 2*a*. Determination of $\nu(t)$.

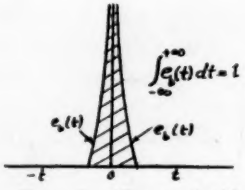


FIG. 2*b*. Shape of $e_b(t)$.

Let the sending e.m.f. $e_0(t)$ be shorted. Let the line be replaced by a resistance $r = \sqrt{L/C}$ and an e.m.f. impulse $e_b(t)$

be introduced in series with $r$ and the lumped network such that (See Fig. 2*b*):

$$e_b(t) = L^{-1}[1] = \delta_0. \tag{15.2}$$

Using the boundary condition for the sending end (4.1) and remembering that $e_0(t) = 0$, we have:

$$D_1 e_a(t) + D_2 i_1(t) = 0. \tag{15.3}$$

But

$$\left.\begin{aligned} e_a(t) &= e_1(t) - e_b(t) \quad (a) \\ \text{and} \qquad i_1(t) &= e_1(t)/r. \quad (b) \end{aligned}\right\} (15.4)$$

Substitute (15.4) in (15.3) and obtain:

$$D_1 e_1 - D_1 e_b + D_2 e_1/r = 0. \tag{15.5}$$

Let $E_1 = L[e_1]$ and use (15.2), then (15.5) transforms as follows:

$$D_1 E_1 - D_1 + D_2 E_1/r = 0 \tag{15.6}$$

or

$$E_1 = \frac{rD_1}{rD_1 + D_2}, \tag{15.7}$$

whence from (15.1*b*)

$$v(t) = L^{-1}[E_1] = e_1(t). \tag{15.8}$$

Thus in order to obtain $v(t)$ experimentally, short the sending e.m.f., replace the line by a resistance $r = \sqrt{L/C}$ and introduce an e.m.f. impulse $e_b(t)$ (See Fig. 2*b*). Then the oscillogram of the voltage drop across $r$ is $v(t)$.

*c. Experimental Determination of $v_s(t)$:* Refer to (15.1*c*) and observe that the only difference between $v(t)$ and $v_s(t)$ is that the network of the receiving end is here used. Hence the experimental determination of $v_s(t)$ is identical with that of $v(t)$ except that the network of the receiving instead of the sending end is used.

16. *Illustrative Problem:* By way of illustrating this treatment consider the distortive line, Fig. 3, and let it be required

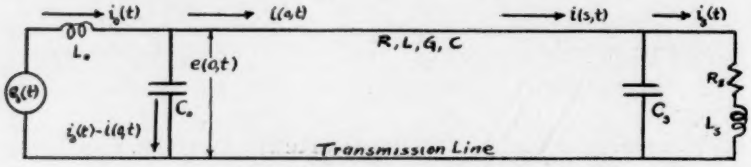


FIG. 3. Transmission Line used in illustrative Problem.

to find $e(x, t)$ and $i(x, t)$ for the case where

$$e_0(t) = \mathbf{1}Ee^{j\omega t}. \tag{16.1}$$

Writing Kirchhoff's Laws for the sending end, Fig. 3, we have:

$$\left.\begin{aligned} e_0(t) &= L_0\frac{di_0(t)}{dt} + \frac{1}{C_0}\int [i_0(t) - i(o, t)]dt, \quad (a)\\ e_0(t) &= L_0\frac{di_0(t)}{dt} - e(o, t). \quad (b)\end{aligned}\right\} (16.2)$$

Eliminating $i_0(t)$ from (16.2) we obtain:

$$[1 + L_0C_0]\frac{d^2}{dt^2}e(o, t) + L_0\frac{di(o, t)}{dt} = e_0(t). \tag{16.3}$$

Comparing (16.3) with (4.1) we have:

$$\left.\begin{aligned} &D_1 = 1 + L_0C_0\frac{d^2}{dt^2}, \quad D_1(u) = 1 + L_0C_0u^2, \quad n_1 = 2, \quad (a)\\ &D_2 = L_0\frac{d}{dt}, \quad\quad D_2(u) = L_0u, \quad\quad n_2 = 1, \quad (b)\\ &D_0 = 1, \quad\quad D_0(u) = 1, \quad\quad n_0 = 0, \quad (c)\end{aligned}\right\} (16.4)$$

$$\left.\begin{aligned} &P_0(u) = 0, \quad \text{because} \quad D_0 = 1 \text{ (See 6.3)}, \quad (a)\\ &E_0(u) = L[e_0(t)] = E(u - j\omega)^{-1}. \quad (b)\end{aligned}\right\} (16.5)$$

Again, writing Kirchhoff's Laws for the receiving end, Fig. 3, we have:

$$\left.\begin{aligned} &e(s, t) = \frac{1}{C_s}\int [i(s, t) - i_s(t)]dt, \quad (a)\\ &L_s\frac{di_s(t)}{dt} + R_si_s(t) - \frac{1}{C_s}\int [i(s, t) - i_s(t)]dt = 0. \quad (b)\end{aligned}\right\} (16.6)$$

Eliminating $i_s(t)$ from (16.6), we have:

$$\left[1 + R_sC_s\frac{d}{dt} + L_sC_s\frac{d^2}{dt^2}\right]e(s, t) - \left[R_s + L_s\frac{d}{dt}\right]i(s, t) = 0. \tag{16.7}$$

Comparing (16.6) with (4.5), we have:

$$\left.\begin{aligned} D_{1s} &= 1 + R_sC_s\frac{d}{dt} + L_sC_s\frac{d^2}{dt^2}, \\ D_{1s}(u) &= 1 + R_sC_su + L_sC_su^2, \end{aligned}\right\} \quad (a)$$

$$D_{2s} = R_s + L_s\frac{d}{dt}, \qquad D_{2s}(u) = R_s + L_su. \quad (b) \qquad (16.8)$$

By Eq. (8.10):

$$\left.\begin{aligned} I_r = L[i_r] &= \frac{E}{(u - j\omega)[r(1 + L_0C_0u^2) + L_0u]} && (a) \\ &= \frac{Ev_0^2}{r(u - j\omega)(u - u_1)(u - u_2)}, && (b) \end{aligned}\right\} \quad (16.9)$$

where

$$\left.\begin{aligned} u_1 &= v_0(-\alpha + \sqrt{\alpha^2 - 1}), && (a) \\ u_2 &= v_0(-\alpha - \sqrt{\alpha^2 - 1}), && (b) \\ v_0 &= 1/\sqrt{L_0C_0}, \qquad r_0 = \sqrt{L_0/C_0}, && (c) \\ \alpha &= \tfrac{1}{2}(r_0/r). && (d) \end{aligned}\right\} \quad (16.10)$$

Assuming that $\alpha \neq 0$ and that $j\omega \neq u_1$, $j\omega \neq u_2$, the inverse of $I_r$ is

$$i_r = E[A_1\epsilon^{u_1t} + A_2\epsilon^{u_2t} + B\epsilon^{j\omega t}], \qquad (16.11)$$

where

$$\left.\begin{aligned} A_1 &= \frac{v_0^2}{r(u_1 - j\omega)(u_1 - u_2)}, && (a) \\ A_2 &= \frac{v_0^2}{r(u_2 - j\omega)(u_2 - u_1)}, && (b) \\ B &= \frac{v_0^2}{r(j\omega - u_1)(j\omega - u_2)}. && (c) \end{aligned}\right\} \quad (16.12)$$

By (10.9) we have

$$\left.\begin{aligned} \nu(t) &= L^{-1}\left[\frac{r(1 + u^2/v_0^2)}{r(1 + u^2/v_0^2) + L_0u}\right] && (a) \\ &= L^{-1}\left[1 - \frac{2\alpha v_0u}{(u - u_1)(u - u_2)}\right] && (b) \\ &= \delta_0 - C_1\epsilon^{u_1t} - C_2\epsilon^{u_2t}, && (c) \end{aligned}\right\} \quad (16.13)$$

where

$$C_1 = 2\alpha v_0 u_1/(u_1 - u_2), \qquad C_2 = 2\alpha v_0 u_2/(u_2 - u_1). \quad (16.14)$$

By (14.5), we have:

$$\left.\begin{aligned} v_s(t) &= L^{-1}\left[\frac{r(1+R_sC_su+L_sC_su^2)}{r(1+R_sC_su+L_sC_su^2)+(R_s+L_su)}\right] && (a)\\ &= L^{-1}\left[1-\frac{R_s+L_su}{r(1+R_sC_su+L_sC_su^2)+(R_s+L_su)}\right] && (b)\\ &= L^{-1}\left[1-\frac{(R_s+L_su)v_s^2}{r(u-u_{1s})(u-u_{2s})}\right], && (c)\end{aligned}\right\} (16.15)$$

where

$$\left.\begin{aligned} u_{1s} &= v_s[-\alpha_{1s}+\sqrt{\alpha_{2s}^2-1}], && (a)\\ u_{2s} &= v_s[-\alpha_{1s}-\sqrt{\alpha_{2s}^2-1}], && (b)\\ v_s &= 1/\sqrt{L_sC_s}, \qquad r_s = \sqrt{L_s/C_s}, && (c)\\ \alpha_{1s} &= \tfrac{1}{2}\left(\frac{r_s}{r}+\frac{R_s}{r_s}\right), \quad \alpha_{2s} = \tfrac{1}{2}\left(\frac{r_s}{r}-\frac{R_s}{r_s}\right). && (d)\end{aligned}\right\} (16.16)$$

Assuming $\alpha_{2s} \neq 1$, we have for the inverse of (16.15$c$):

$$v_s(t) = \delta_0 - C_{1s}\epsilon^{u_{1s}t} - C_{2s}\epsilon^{u_{2s}t}, \quad (16.17)$$

where

$$C_{1s} = \frac{v_s^2(R_s+L_su_{1s})}{r(u_{1s}-u_{2s})}, \qquad C_{2s} = \frac{v_s^2(R_s+L_su_{2s})}{r(u_{2s}-u_{1s})}. \quad (16.18)$$

We now proceed to compute the convolution integrals occurring in (10.13), (10.14) and (14.8). In all cases we use the fact that:

$$\left.\begin{aligned} \epsilon^{at}*\epsilon^{bt} &= (\epsilon^{at}-\epsilon^{bt})/(a-b) \quad \text{if} \quad a \neq b, && (a)\\ \epsilon^{at}*\epsilon^{at} &= t\epsilon^{at}, && (b)\end{aligned}\right\} (16.19)$$

Using relations (16.19) and substituting (16.11), (16.13$c$) in (10.14) we have

$$i_r*\epsilon^{-tG/C} = E[a_1\epsilon^{u_1t} + a_2\epsilon^{u_2t} + b\epsilon^{j\omega t} + c\epsilon^{-tG/C}], \quad (16.20)$$

where

$$\left.\begin{aligned} a_1 &= A_1/(u_1+G/C), \qquad b = B/(j\omega+G/C), && (a)\\ a_2 &= A_2/(u_2+G/C), \qquad c = -(a_1+a_2+b), && (b)\end{aligned}\right\} (16.21)$$

$$i_{01} = i_r*\epsilon^{-tG/C}*v$$

$$= E[a_{01}\epsilon^{u_1t} + a_{02}\epsilon^{u_2t} + b_0\epsilon^{j\omega t} + c_0\epsilon^{-tG/C}], \quad (16.22)$$

where

$$\left.\begin{aligned} a_{01} &= a_1 - \frac{C_1a_2+C_2a_1}{u_1-u_2} - \frac{C_1c}{u_1+G/C} - \frac{C_1b}{u_1-j\omega} - C_1a_1t, && (a)\\ a_{02} &= a_2 - \frac{C_1a_2+C_2a_1}{u_2-u_1} - \frac{C_2c}{u_2+G/C} - \frac{C_2b}{u_2-j\omega} - C_2a_2t, && (b)\\ b_0 &= b + \frac{C_1b}{u_1-j\omega} + \frac{C_2b}{u_2-j\omega}, && (c)\\ c_0 &= c + \frac{C_1c}{u_1+G/C} + \frac{C_2c}{u_1+G/C}, && (d) \end{aligned}\right\} \quad (16.23)$$

$$i_r * \epsilon^{-\rho t} = E(m_1\epsilon^{u_1 t} + m_2\epsilon^{u_2 t} + n_1\epsilon^{j\omega t} + p\epsilon^{-\rho t}), \quad (16.24)$$

where

$$\left.\begin{aligned} m_1 &= A_1/(u_1+\rho), \qquad m_2 = A_2/(u_2+\rho), && (a)\\ n &= B/(j\omega+\rho), \qquad p = -(m_1+m_2+n). && (b) \end{aligned}\right\} \quad (16.25)$$

The substitution of (16.22) and (16.24) in (10.13) gives expressions for $e_\alpha$ and $i_\alpha$ in which numerical values can be directly used.

Using relations (16.19) and substituting (16.13) in (14.8) we have:

$$(\nu - \delta_0) * \epsilon^{-tG/C} = g_1\epsilon^{u_1 t} + g_2\epsilon^{u_2 t} + h\epsilon^{-tG/C}, \quad (16.26)$$

where

$$g_1 = -C_1/(u_1+G/C), \qquad g_2 = -C_2/(u_2+G/C), \qquad h = -(g_1+g_2), \quad (16.27)$$

$$(\nu - \delta_0) * \epsilon^{-tG/C} * \nu = g_{01}\epsilon^{u_1 t} + g_{02}\epsilon^{u_2 t} + h_0\epsilon^{-tG/C}, \quad (16.28)$$

where

$$\left.\begin{aligned} g_{01} &= g_1 - \frac{C_1g_2+C_2g_2}{u_1-u_2} - \frac{C_1h}{u_1+G/C} - C_1g_1t, && (a)\\ g_{02} &= g_2 - \frac{C_2g_1+C_1g_2}{u_2-u_1} - \frac{C_2h}{u_2+G/C} - C_2g_2t, && (b)\\ h_0 &= h + \frac{C_1h}{u_1+G/C} + \frac{C_2h}{u_2+G/C}. && (c) \end{aligned}\right\} \quad (16.29)$$

The substitution of (16.28) in (14.8*a*) yields an expression for $k_0$ in which numerical values can be directly used.

If in $k_0$ we replace $C_1$, $C_2$, $u_1$, $u_2$ by $C_{1s}$, $C_{2s}$, $u_{1s}$, $u_{2s}$ respectively, we obtain an expression for $k_s$ in which numerical values can be directly used.

We now have all the terms required for substitution in (12.2) and determining $e$ and $i$.

## APPENDIX.

### Transformation Rules and Theorems.

The following transformation rules and theorems are used in the paper.

*a. Laplace Transform of Derivatives:* It can be shown [13] that if:

$$Df(t) = \sum_{k=0}^{n} a_k f^{(k)}(t), \tag{A.1}$$

then

$$\left.\begin{aligned} L[Df(t)] &= \sum_{k=1}^{n} [A_{k1} + uA_{k2} + u^2A_{k3} + \cdots \\ &\qquad + u^{k-1}A_{kk}] + D(u)\cdot L[f(t)], \quad (a) \\ \text{where} \quad A_{ks} &= a_k f^{(k-s)}(o), \qquad f^{(m)}(o) = \lim_{t\to 0+} f^{(m)}(t), \\ D(u) &= \sum_{k=0}^{n} a_k u^k. \qquad (b) \end{aligned}\right\} (A.2)$$

*b. Convolution Theorem:* This theorem [12p163*] is often called Borel's Theorem in the literature and states:
If

$$\left.\begin{aligned} &L^{-1}[F(u)] = f(t), \qquad L^{-1}[G(u)] = g(t), \\ &\qquad L^{-1}[F(u)G(u)] = h(t) \text{ continuous}, \quad (a) \\ &\text{then} \\ &h(t) = L^{-1}[F(u)G(u)] \\ &\qquad = \int_0^t f(\tau)g(t-\tau)d\tau = \int_0^t f(t-\tau)g(\tau)d\tau. \quad (b) \end{aligned}\right\} (A.3)$$

The integral to the right is called the " convolution " in-

* Note: "12p163" means reference 12, page 163.

tegral and is denoted as follows throughout this paper:

$$[f*g]_t = \int_0^t f(\tau)g(t-\tau)d\tau = \int_0^t f(t-\tau)g(\tau)d\tau. \quad (A.4)$$

For the repeated convolution integrals we use the power notation as follows

$$\left.\begin{aligned} [f^{*1}]_t &= f(t), && (a)\\ [f^{*2}]_t &= [f*f]_t, && (b)\\ [f^{*k}]_t &= [f^{*(k-1)}*f]_t, \quad k = 2, 3 \cdots. && (k) \end{aligned}\right\} (A.5)$$

Note: If $F(u)$ and $G(u)$ are known to be transforms of $f(t)$ and $g(t)$ respectively, but $F(u)\cdot G(u)$ is not, then, [12p164] at least $\frac{1}{u^2}F(u)G(u)$ is a transform namely

$$\frac{1}{u^2}F(u)G(u) = L[t*f*g]. \quad (A.6)$$

*c. Criteria for Transforms:* The following criteria enable us to ascertain, in some cases, whether a given function $F(u)$ is the transform of an $L$-function $f(t)$. In all the criteria it is assumed that $F(u)$ is analytic in some right half plane for otherwise it cannot be the transform of an $L$ function.[12p43]

(1) If $F(u)$ is of the form

$$F(u) = \frac{F_1(u)}{u^{1+\delta}}, \quad \delta > 0, \quad F_1(u) \text{ bounded}, \quad (A.7)$$

then $F(u)$ is the transform [12p128] of an $L$-function $f(t)$ which is continuous for $t \geqq 0$.

(2) If $G(u) = \sum_{k=0}^{m} a_k u^k / \sum_{k=0}^{n} b_k u^k, \quad m < n, \quad b_n \neq 0, \quad (A.8)$

then $G(u)$ is the transform of a continuous $L$-function $g(t)$ which is found by expanding $G(u)$ into a sum of partial fractions and inverting each of the fractions [13.12p323] (Expansion Theorem).

(3) If $F(u) = H(u) \sum_{k=0}^{m} a_k u^k / \sum_{k=0}^{n} b_k u^k,$

$$m \leqq n, \quad b_n \neq 0, \quad (A.9)$$

where $H(u)$ is the transform of the $L$-function $h(t)$, then $F(u)$ is the transform of an $L$-function $f(t)$. To show that such is the case simply expand the rational function and multiply each term by $H(u)$ obtaining a sum of terms each of which is the transform of an $L$-function.

$$(4)\quad \text{If } H(u) = F(u)\sum_{k=0}^{m} a_k u^k \Big/ \sum_{k=0}^{n} b_k u^k,$$

$$n \leqq m, \quad H(u) = L[h(t)], \qquad (A.10)$$

then each of the functions $u^r F(u)$, $r = 0, 1, 2 \cdots$, $n - m$ is a transform, moreover, there is a function $f(t)$ such that:

$$u^r F(u) = L\left[\frac{d^r f(t)}{dt^r}\right], \quad r = 0, 1, 2 \cdots n - m. \qquad (A.11)$$

*Proof:* Write $(A.10)$ in the following form:

$$u^r F(u) = H(u) u^r \sum_{k=0}^{n} b_k u^k \Big/ \sum_{k=0}^{m} a_k u^k. \qquad (A.12)$$

Since $r \leqq n - m$, $H(u)$ in $(A.12)$ is multiplied by a rational function as in $(A.9)$. Hence, by (3), $u^r F(u)$ is a transform. Write:

$$u^{n-m} F(u) = G(u) = L[g(t)]. \qquad (A.13)$$

Again, by (3),

$$F(u) = u^{-(n-m)} G(u) = L[f(t)]. \qquad (A.14)$$

Now if $(n - m) \geqq 1$, then by the convolution theorem (See $b$):

$$\left.\begin{aligned} f(t) &= L^{-1}[u^{-(n-m)}] * L^{-1}[G(u)] \\ &= \frac{t^{n-m-1}}{(n-m-1)!} * g(t), \qquad (a) \\ &= \frac{1}{(n-m-1)!}\int_0^t (t-\tau)^{n-m-1} g(\tau)\, d\tau. \qquad (b) \end{aligned}\right\} (A.15)$$

Now from $(A.15b)$ it is obvious that $f(t)$ has $n - m$ derivatives for $t > 0$ and that

$$\lim_{t \to 0+} \frac{d^r f(t)}{dt^r} = 0, \quad r = 0, 1, 2 \cdots n - m - 1. \qquad (A.16)$$

Hence, by $(A.2)$, Eq. $(A.11)$ is proved.

*d. Extension of the Convolution Theorem:* Although we have shown, in $c(4)$, that $u^r F(u)$, $r = 0, 1, 2 \cdots n - m$, is a transform, its inverse cannot be written as a convolution integral because $u^r$ is not the transform of an $L$-function. This fact causes formal complications which can be avoided by extending the convolution theorem. To do this we define $u^r$ as the transform of the symbolic function $\delta_r(t)$:

$$u^r = L[\delta_r(t)], \quad r = 0, 1, 2 \cdots. \tag{A.17}$$

We agree to use the function $\delta_r(t)$ only in convolution integrals where it would acquire the following meaning:

$$[\delta_r * f]_t = \frac{d^r f}{dt^r} \text{ for any } L\text{-function } f(t) \text{ with } r \text{ derivatives.} \tag{A.18}$$

We shall now show that the introduction of the symbolic function enables us to extend the convolution theorem to any products of the form $H(u) = G(u)F(u)$ where $G(u)$ is a rational function (proper or improper). For let:

$$\left.\begin{aligned} &F(u) = L[f(t)], \qquad H(u) = L[h(t)], && (a)\\ &G(u) = \sum_{r=0}^{k} c_r u^r + G_1(u) \text{ where } G_1(u) \text{ is a proper} \\ &\quad \text{rational fraction such that } G_1(u) = L[g_1(t)]. && (b) \end{aligned}\right\} \tag{A.19}$$

Then, by definition $(A.17)$, $G(u)$ would have the symbolic inverse $g(t)$:

$$g(t) = \sum_{r=0}^{k} c_r \delta_r(t) + g_1(t). \tag{A.20}$$

In order to show that $[g*f]_t = h(t)$, use $c(4)$ and $(A.18)$ and obtain:

$$\left.\begin{aligned} h(t) &= \sum_{r=0}^{k} c_r \frac{d^r f(t)}{dt^r} + [g_1 * f]_t, && (a)\\ &= \sum_{r=0}^{k} c_r [\delta_r * f]_t + [g_1 * f]_t, && (b)\\ &= [g*f]_t, \quad \text{by } (A.20). && (c) \end{aligned}\right\} \tag{A.21}$$

This extension of the convolution theorem enables one to speak of the inverse of a rational function $G(u)$ irrespective

whether $G(u)$ is a proper or improper fraction provided the inverse of $G(u)$ is used only in connection with the convolution theorem.

*e. Transfer Theorem:*

If: (1) the $L$-function $f(t)$ has $n + 1$ derivatives for $t > 0$, $n \geqq 0$, and

$$\lim_{t \to 0+} f^{(k)}(t) = 0, \quad k = 0, 1, 2 \cdots. \tag{A.22}$$

(2) $\Phi(u)$ is analytic in some half plane and has the form

$$\Phi(u) = \frac{C}{(u + \beta)^{\delta_1}} + \frac{\Phi_1(u)}{u^{1+\delta_2}},$$

$$\delta_1 > 0, \quad \delta_2 > 0, \quad \Phi(u) \text{ bounded.} \tag{A.23}$$

Then:

(1) $G(u) = L[\phi]L[f(t)] = \Phi F(u)$ is the Laplace transform of an $L$-function $g(t)$ which has $n + 1$ derivatives for $t > 0$.

$$(2) \lim_{t \to 0+} g^{(k)}(0) = 0, \quad k = 0, 1, 2 \cdots n. \tag{A.24}$$

*Proof:* By $c(1)$, $\Phi_1(u)/u^{1+\delta_2}$ is the Laplace transform of a continuous function $\phi_1(t)$. Hence:

$$\left.\begin{aligned} &\Phi(u) = L[\phi(t)], && (a)\\ &\text{where (see } A.23) \\ &\phi(t) = \frac{C}{\Gamma(\delta_1)} \epsilon^{-\beta t} t^{\delta_1 - 1} + \phi_1(t). && (b) \end{aligned}\right\} (A.25)$$

Since $\delta_1 - 1 > -1$

$$\left.\begin{aligned} &\psi(t) = \int_0^t \phi(\tau) d\tau \quad \text{exists} && (a)\\ &\text{and} \\ &\lim_{t \to 0+} \psi(t) = 0. && (b) \end{aligned}\right\} (A.26)$$

Define a function $g(t)$ as follows:

$$g(t) = [f * \phi]_t = \int_0^t f(t - \tau)\phi(\tau) d\tau. \tag{A.27}$$

Integrate ($A.27$) by parts and substitute ($A.26$), obtaining:

$$\left.\begin{aligned} g(t) &= [f(t-\tau)\psi(\tau)]^t - \int_0^t f'(t-\tau)\psi(\tau)d\tau \quad (a) \\ &= -\int_0^t f'(t-\tau)\psi(\tau)d\tau. \quad (b) \end{aligned}\right\} (A.28)$$

Differentiating ($A.28b$) $n$ times with respect to $t$ and using hypothesis (1), we obtain:

$$\left.\begin{aligned} g^{(k)}(t) &= -\int_0^t f^{(k+1)}(t-\tau)\psi(\tau)d\tau \\ & \qquad\qquad k = 0, 1, \cdots n, \quad (a) \\ \lim_{t\to 0+} g^{(k)}(t) &= 0 \qquad\qquad k = 0, 1, \cdots n. \quad (b) \end{aligned}\right\} (A.29)$$

In order to show that $g(t) = L^{-1}[G(u)]$ we observe that by the note to the convolution theorem:

$$\left.\begin{aligned} \frac{1}{u^2}F(u)\phi(u) &= L[g_1], \quad (a) \\ \text{where} \\ g_1 = t*f*\phi &= t*g = \int_0^t (t-\tau)g(\tau)d\tau. \quad (b) \end{aligned}\right\} (A.30)$$

Differentiate ($A.30b$) and obtain

$$g'_1 = \int_0^t g(\tau)d\tau, \qquad g''_1 = g(t). \qquad (A.31)$$

Now from ($A.30$) and ($A.31$) we have:

$$g_1(0) = 0, \qquad g'_1(0) = 0. \qquad (A.32)$$

Hence, by ($A.2$):

$$\left.\begin{aligned} L[g] = L[g''_1] &= u^2\frac{1}{u^2}F(u)\Phi(u), \quad (a) \\ &= F(u)\Phi(u) = G(u). \quad (b) \end{aligned}\right\} (A.33)$$

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**How Old the Earth.**—*The Industrial Bulletin* of Arthur D. Little Inc., No. 181, states that a planetarium show depicting the various ways that the earth might come to its end makes it seem that the earth cannot long continue to survive the logically and vividly presented hazards. Yet, when a geologist talks in terms of the millions of years of history already experienced by the earth, the risk of extinction in the next hundred-, thousand- or even ten million-year period seems small. Radioactive elements decompose spontaneously at a fixed rate, dependent upon the nature of the particular element to produce substances that can be measured with accuracy. Uranium and thorium both decompose with extreme slowness, but at a known rate, into the stable substances lead and helium. Determinations made today of the relative proportions of the lead and uranium in a mineral yield, by calculation, the number of years required for uranium to produce the observed lead, provided there is no other source of lead. In other minerals, it is convenient and accurate to release and measure the helium gas imprisoned within and compare that with the content of uranium or other radioactive substances that give rise to it. Measurements by careful investigators indicate that the oldest rocks to be found on earth are of the order of 1,800,000,000 years, and the oldest in meteorites, which strike the earth from outside, some 2,800,000,000 years. These ages date not from the origin of these bodies but from the crystallization of the minerals that form the rocks. Previously, the mass from which the minerals crystallized must have passed through a long period in a molten condition. Other possibilities exist besides the "radioactive time-clock" for going back into the reaches of time. The moon-made tides on the earth sap the energy, causing a slowing-down of the earth and a retreat of the moon from the earth. Astronomers calculate, from the present distance of the moon and its rate of slowing, that it must have revolved around the earth for about 4,000,000,000 years. It is considered conservative to think of the earth as perhaps three or four billion years old and of the sun as even more venerable.

R. H. O.

## NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

### RUBBER LABORATORY IN BRAZIL.

In an effort to develop its natural resources to the highest point, the Government of Brazil is establishing several regional agricultural institutes; one of these is located at Belem, Para. Through the State Department, the Bureau was requested to assist the Brazilian Government in equipping this laboratory and developing a research program.

Under a provision of an Act of Congress which provides for the loan of technical experts to South and Central American countries, Dr. Norman Bekkedahl of the Bureau's rubber section has gone to Belem where he will serve as chief of the Rubber Technology Division of the Instituto Agronomico do Norte. He has, as his assistant, Fred L. Downs, formerly rubber research chemist on the staff of the American Steel and Wire Company. Before leaving the United States, Dr. Bekkedahl placed orders for the equipment needed to set up a modern rubber research and testing laboratory. When completed, it will be the only one of its kind serving the United Nations in the tropics and at the source of the rubber supply.

Dr. Bekkedahl is well qualified for this important post. He has done a great deal of work on the physico-chemical and thermodynamic properties of rubber, as described in previous numbers of this Bulletin. In 1938 he was appointed by the President to act as delegate for the United States at the Tenth International Congress of Chemistry, at Rome. He served as delegate of the National Academy of Sciences at the same conference and at the Thirteenth Conference of the International Union of Chemistry. He represented the Bureau at the International Rubber Technology Conference in London.

Mr. Downs served as a student assistant in the Bureau's rubber section and as chemist with the Thermoid Rubber

* Communicated by the Director.

Company, where he became familiar with testing and control work. In 1935 he joined the staff of the electrical cable division of the American Steel and Wire Company, and was concerned with the development and testing of rubber insulation. He has had wide experience in the handling of synthetic rubbers and plastic materials.

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### DENSITIES OF SYNTHETIC RUBBERS.

Synthetic rubber can be produced under conditions which are much better known and more subject to control than those involved in the production of natural rubber. It is logical, then, to measure its density with greater precision, in the hope of being able to ascribe significance to the variations found when one sample is compared with another. The densities of different samples of natural rubber show variations as great as 1 or 2 per cent., and have never been correlated with the origin or subsequent treatment of the rubber.

The precise measurement of densities of synthetic rubbers forms the subject of a paper by Lawrence A. Wood, Norman Bekkedahl, and Frank L. Roth in the November number of *Industrial and Engineering Chemistry* and which will also appear as RP1507 in the December *Journal of Research*. The paper gives the method of preparing specimens in suitable form, the details of the actual measurements by the method of hydrostatic weighings, and the results obtained for 18 varieties of synthetic rubber, including almost all of those now manufactured in the United States.

Samples of synthetic rubbers as received from the manufacturer were never found to be in a form suitable for precise measurements of the density. Even where there appeared to be no entrapped air and the surface seemed to be relatively smooth, molding produced specimens yielding higher and much more consistent values. A vacuum chamber was designed so that the specimen could be heated and molded under vacuum into a sheet about 1/16 inch thick. The density of specimens weighing about 1 gram each, which had been cut from this sheet, was determined. The values obtained with different specimens from the same sample rarely differed from each other by more than 0.05 per cent. Measurements were

made soon after molding because some rubbers recover and develop roughened surfaces and vacuoles which bring about a decrease in the apparent density.

Unvulcanized Buna S, prepared in a laboratory polymerization with a minimum quantity of materials other than butadiene and styrene, was found to have a density at 25° C. of 0.9291 g./cm$^3$. Corresponding values for butadiene-styrene copolymers produced on a commercial scale were as follows: Firestone Buna S, 0.9358; U. S. Rubber Co. Buna S, 0.9369; Standard Oil (N. J.) Buna S, 0.9390; Chemigum IV, 0.9391; Hycar O.S.-20, 0.9385; and Hycar O.S.-30, 0.9303 g./cm$^3$. The densities of other common varieties of synthetic rubber were found to be as follows: Neoprene CG, 1.2307; Neoprene E, 1.2384; Neoprene FR, 1.1406; Neoprene GN, 1.2290; Chemigum I, 1.0135; Hycar O.R., 0.9992; Perbunan, 0.9684; Thiokol RD, 1.0564; Thiokol A, 1.5983; Thiokol FA, 1.3298; and Butyl B-1.45, 0.9175 g./cm$^3$.

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### COMBINATION OF WOOL WITH ACIDS IN MIXTURES.

A theory of dyeing should explain the nature of the attraction of dyes for textile fibers, how the dyes go on, and what makes them stick so that they resist washing. It should also aid in understanding the effect on the transfer of the dye from bath to fibers of such factors as the amounts of each acid, dye, and salt present in a complicated solution, the dyebath.

Papers already published have provided a basis for such a theory by demonstrating that different strong acids, including acid dyes, combine with wool to very different extents. These differences have been interpreted in terms of the combination of wool with the anions of acids as well as with hydrogen ions, the combination with each anion being governed by its own distinct and characteristic affinity. In Research Papers RP1510 and RP1511 in the December number of the *Journal of Research*, Jacinto Steinhardt, Charles N. Fugitt, and Milton Harris, Research Associates of the Textile Foundation, extend this analysis on the basis of measurements of the amounts of each of *two* acids which combine with wool when a dye acid or salt and various second acids are present in mixtures. It is shown that the two anions compete with

each other for combination with the fibers, so that the amounts of each combined depend not only on the amounts initially present but on their respective affinities for wool, as previously determined.

The bearing of these results on the theory of acid dyeing is discussed, with special reference to the factors promoting the attainment of level and solid application of dye to the fibers.

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### PHASE-EQUILIBRIUM STUDIES INVOLVING POTASH COMPOUNDS OF PORTLAND CEMENT.

In the December *Journal of Research* (RP1512), William C. Taylor describes an investigational program designed to determine the manner in which $K_2O$ is combined in portland cement clinker. In general, this involves a series of phase-equilibrium studies of binary and ternary systems. The systems to be included are chosen, from time to time, by examining the products of reaction between a known clinker compound and a newly established compound in a system just studied. Specifically, the research on $K_2O$ in clinker has been conducted as follows: The compound $K_2O.Al_2O_3$ was shown to exist in the system $K_2O$-$CaO$-$Al_2O_3$; following studies showed that $K_2O.Al_2O_3$ and $4CaO.Al_2O_3.Fe_2O_3$ form a simple binary system with no additional compound; in ensuing studies, however, it was found that $K_2O.Al_2O_3$ reacts with $3CaO.SiO_2$ and with $2CaO.SiO_2$ in the formation of a ternary potash-lime-silica compound; the composition of this potash-lime-silica compound was established as $K_2O.23CaO.12SiO_2$ by a study of the system $K_2O.CaO.SiO_2$-$2CaO.SiO_2$; this compound was shown to be stable in the presence of $3CaO.Al_2O_3$ and $4CaO.Al_2O_3.Fe_2O_3$; the present study of the system $K_2O.23CaO.12SiO_2$-$CaO$-$5CaO.3Al_2O_3$ was then undertaken to determine if any other potash compound might exist in clinkers composed of $K_2O$, $CaO$, $Al_2O_3$ and $SiO_2$, and comparable to portland cement clinker in composition.

The ternary system $K_2O.23CaO.12SiO_2$-$CaO$-$5CaO.3Al_2O_3$ was investigated and a phase diagram constructed. The only solid phase, other than the components observed in the system, was the compound $3CaO.Al_2O_3$.

Studies of the products of reaction between $K_2O.23CaO.12SiO_2$ and $CaSO_4$, and also of those produced when clinker raw mixtures containing both $K_2O$ and $SO_2$ are heated, showed that $K_2O$ combines preferentially with $SO_3$ to form $K_2SO_4$. It was found that any $K_2O$ in excess of the amount required by the $SO_3$ in the formation of $K_2SO_4$, combined with CaO and $SiO_2$ to form $K_2O.23CaO.12SiO_2$. No evidence was obtained of any potash compounds other than $K_2SO_4$ and $K_2O.23CaO.12SiO_2$ in clinkers composed of $K_2O$, CaO, $Al_2O_3$, $Fe_2O_3$, $SiO_2$ and $SO_3$.

At elevated temperatures, the molten $K_2SO_4$ appeared to be immiscible with the other liquid phase formed from the components of the clinker and, consequently, seemed to have no effect on the products of crystallization. Upon cooling, $K_2SO_4$ crystallized extremely rapidly.

Crystals of $K_2SO_4$ were identified in several commercial portland cement clinkers.

No reaction of either $K_2SO_4$ or $K_2O.23CaO.12SiO_2$ with MgO was observed, which indicates that the presence of MgO does not affect the manner in which $K_2O$ is combined in clinker.

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### SUBSTITUTES FOR METAL IN HEATING DUCTS.

Restrictions against the use of metal, particularly galvanized and tinned sheets, make necessary the use of substitutes. Unfortunately no other readily available material has the same characteristics from the view point of the sheet metal worker. A number of fire tests with proposed substitutes have been recently conducted at the Bureau.

A substitute that can be worked with the same tools and technique would find more ready acceptance by the trade, because any changes required in the procedure of fabrication and erection will usually work a hardship by slowing up the work. For these reasons the thin pliable sheets have found favor although some of the rigid boards may have superior structural strength and durability. Since fire safety requirements are lower for returns than for the hot air ducts, different materials for the two kinds of ducts are used in many installations.

Among the pliable boards offered are several composed of a felt base with high melting point asphalt saturant and coating. Others are of similar composition for the core with facings of asbestos paper. These are combustible. Other pliable boards have paper boxboard or pressboard cores with asbestos paper facings. Some of these are rated "Combustible" or "Slow-burning," and others "Fire Retardant," as tested according to the procedure outlined in Federal Specification SS-A-118 for prefabricated acoustical units, to which reference is made for testing methods and classification requirements. The incorporation of bituminous materials to lower absorption will usually increase flammability. Fire retardant treatment of pressboard or boxboard cores makes for increased safety from flame spread.

Cement-asbestos boards have many characteristics that make them suitable for duct walls, although they are not readily worked into turns, branches, and other complicated fittings. In one system of fabrication, the edges of the board are mitered and cemented together with an adhesive, after which paper tape is applied to bind the corners on the outside. Another system employs grooved strips of flameproofed wood for corner members, and a third employs metal corner strips. All of these require some metal for fittings and unusual conditions.

A substitute for galvanized metal suitable for ducts or duct fittings recently placed on the market consists of pickled iron sheets coated with a high-melting point compound of fatty acid pitch containing a high percentage of inert mineral filler.

This coating is covered with mica dust which gives the sheet a gray color and a greasy feel. The technique and tools for working this material are the same as for galvanized metal except that preparation for soldering requires the removal of the coating from the areas to be joined. It is probable that this material will be acceptable to sheet metal workers for use in lieu of the galvanized steel usually employed. The tests have indicated that only materials classed as "Incombustible" or "Fire Retardant" should be used for hot air supply ducts, with those of the latter classification at not less than 6 ft. from the heating plant if liquid or solid

fuels are employed. Material for return ducts may be of lower classification with respect to fire safety, but combustible materials should not be used within 3 ft. of the heating plants.

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### OXYGEN REQUIREMENTS AT HIGH ALTITUDES.

Normal persons become markedly less efficient at altitudes of 10,000 to 15,000 feet and collapse at around 20,000 feet when breathing air, but when breathing supplemental oxygen, they can ascend to much greater altitudes without danger of collapse.

In an article by W. A. Wildhack in the December number of the *Journal of the Aeronautical Sciences*, theoretical equations are developed to determine what concentrations of oxygen must be breathed at any altitude to maintain the "oxygen equivalent" of air at selected altitudes—i.e., the same alveolar oxygen pressure. The partial pressures of water vapor and carbon dioxide are taken into account, as well as the variation of the pressure of carbon dioxide with the alveolar oxygen pressure.

Six curves are given on a chart showing the oxygen concentrations required at any altitude to maintain the oxygen equivalent of air at sea level, and at altitudes of 5, 10, 15, 20, and 25 thousand feet. Altitudes of 33.8, 40, and 46 thousand feet with pure oxygen are equivalent to air at sea-level, 10 and 20 thousand feet, respectively.

It is shown that these equivalents are nearly independent of the assumptions as to the state of physiological adaptation, and should be applicable to all individuals.

**Ameripol—Synthetic Rubber Developed by U. S. Scientists.**—C. C. PRYOR. (*The Petroleum Engineer*, Vol. 13, No. 12.) One of the largest plants in the United States engaged in the commercial manufacture of butadiene-type synthetic rubbers is operated by the Hycar Chemical Company, a firm jointly owned by Phillips Petroleum Company and the B. F. Goodrich Company. The plant was originally designed and constructed by Goodrich to operate under patents and techniques developed by that company. The plant is operating approximately at its full capacity at the present time. DR. WALDO L. SEMON, manager of Goodrich's synthetic research division, and discoverer of Ameripol, first experimented with the manufacture of dienes and their polymerization to produce synthetic rubber in 1926. It has been under the direction of Semon that the development and manufacture of Ameripol and other butadiene synthetic rubbers by B. F. Goodrich have been brought to a commercial basis. The first pilot plant by the company was completed in 1939 with a capacity of 100 lb. of synthetic rubber per day. By 1940 two distinct types of butadiene copolymer synthetic rubber had been fully developed by the company. The first of the rubbers was a tire rubber and the second an oil-resistant specialty rubber, neither of which infringed on the German Buna rubber patents. Each of two synthetics has been marketed under the trade name Ameripol, signifying the American polymer. A commercial synthetic rubber plant was constructed with a capacity of about 2,000 long tons per year to manufacture both types of the synthetic rubbers developed by the company. The same equipment and almost identically the same technique are employed in compounding and vulcanizing the Ameripol rubber as are used for natural crude rubber. The physical properties of Ameripol rubbers are in general superior to those of natural rubber. Most important among these physical properties of Ameripol are its tensile strength and elongation. Due to the fact that the Ameripol rubbers are compounded differently for each of a large number of special uses, the strength and elasticity of the compounds vary over a wide range. Compounds having tensile strengths greater than 4,000 lb. per sq. in. and elongations of more than 600 per cent. are, however, easily manufactured.

R. H. O.

# THE FRANKLIN INSTITUTE

## STATED MONTHLY MEETING, DECEMBER 16, 1942

The stated monthly meeting of The Franklin Institute was called to order at 8:15 P.M. by Mr. Charles S. Redding, President, after the playing of the national anthem.

Upon motion the minutes of the November meeting were approved as printed in the December number of the JOURNAL of the Institute.

The Chairman then called upon the Secretary, Dr. Henry Butler Allen, for a report.

Dr. Allen stated that since the last meeting the following additions had been made to the membership:

| | |
|---|---|
| Active Family | 3 |
| Sustaining | 3 |
| Active | 84 |
| Active Non-Resident | 5 |
| Associate Family | 2 |
| Associate | 22 |
| Associate Library | 1 |
| Student Library | 7 |
| Student | 14 |
| Total | 141 |

He then announced that the annual Christmas Week lectures, on the James Mapes Dodge Foundation, would be held on December 28, 29 and 30, and that the lecturer was John R. Roberts, of the University of Pennsylvania, who had chosen for his subject, "Your Place in the World of Tomorrow." He pointed out that tickets for this series would make fine Christmas gifts and could be procured at the office.

He extended a cordial invitation to members and friends to attend the Franklin Birthday Celebration to be held at noon on Saturday, January 16, in Franklin Hall, stating that it would be a joint ceremony with the Poor Richard Club.

Dr. Allen then called attention to the fact that nominations of officers for the ensuing year, together with one-third of the Board of Managers, were in order at this meeting, in accordance with Article IV, Section 4, of the By-Laws of The Franklin Institute.

In compliance with regulations, the following names were submitted:

*President*
(to serve one year).
Charles S. Redding

*Vice-Presidents*
(to serve one year).
Walton Forstall
W. Chattin Wetherill
Samuel S. Fels
Richard W. Lloyd
Richard T. Nalle

*Treasurer*
(to serve one year).
M. M. Price

*Members of the Board of Managers*
(to serve three years).
Irenee duPont
W. H. Fulweiler
Nathan Hayward
Horace P. Liversidge
Malcolm Lloyd, Jr.
Marshall S. Morgan
Edward Warwick
D. Robert Yarnall

(to serve two years).
Hiram S. Lukens

(to serve one year).
James H. Robins

The Chairman then called for nominations from the floor.

There being no other names submitted, upon motion, duly seconded, the nominations were declared closed.

Mr. Redding then introduced Dr. Alexander Silverman, Professor of Chemistry, University of Pittsburgh, as the speaker of the evening. Dr. Silverman spoke on "Glass and the War," explaining what could be told of the part of glass in the war today. He pointed out the use of various types: plate glass, optical glass, fiber glass, tubing and special illuminating units, emphasizing the benefits which have been derived through the perfection of special types. The lecture was illustrated with specimens, motion pictures and lantern slides.

The meeting was dismissed with a rising vote of thanks to the lecturer for a very interesting evening.

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## NEW MEMBERS OF THE FRANKLIN INSTITUTE ELECTED, JAN. 1 TO NOV. 30, 1942.

### ACTIVE LIFE MEMBER.

Holstein DeHaven Fox, Jr.

### SUSTAINING MEMBERS.

I. L. Couch
Ralph Kearney
Carl H. Lenz
Hermann Maack
Lionel P. Maggenti
E. J. Ryan
William T. Walker

### ACTIVE MEMBERS WITH FAMILY PRIVILEGES.

Ernest R. Aemisegger
Wayne MacV. Barker
John M. Beatty
Corliss A. Bercaw
Willis L. Bissinger
Johan Bjorksten
Rodney T. Bonsall
Claude N. Campbell
Frank C. Coe
Frederick Cooper
Harold J. Cowell
Leo J. Dawson

Joseph P. Dunn
Charles H. Evoy, Jr.
J. Russell Fawley
A. L. Fitzpatrick
Davidson R. Fleming
William Franz
E. H. Grafton
Russell B. Graves
Albert J. Grosser
John Hamilton
Milton I. Hartman
William F. Hawk
Walter Woodward Hawsey
M. G. Herbach
John B. Janney
Henry J. Kaltenthaler
Louis Kasper
Harry Keller
James V. Kelsall
Burns Lafferty
Russell Lake
W. Palmer Lanagan
David S. Levitan
Christian J. Ludwig
Charles Matthias
John McElgin
F. P. McNinch
John S. McQuade
Marvin C. Moffett
Roland S. Morris
Lester Myers
Harry Lloyd Nelson
F. Price Norris, Jr.
George F. Pettinos, Jr.
Mrs. Bess K. Potter
Stanley Pozelewski
Mrs. Howard C. Pruyn
Alexander L. Pugh, Jr.
Joseph Richter
Isaac B. Ritter
Victor I. Roystuart
Mrs. Julia Sammartino
Jose I. Saraiva
Albert E. Scheinert
T. W. Schmidt
Walter Schoeni
William Schwarze, Jr.
Albert B. Seither, Jr.
Samuel Shapiro
Herman A. Shelanski
Wilbur W. Stranahan
John L. Stringer
George E. Swift
Thomas T. Taylor
M. L. Thompson
John Trautinger, Jr.
Morton Tucker
John L. Vanderherchen
John Watt
J. William Wetter
Sydney K. Wilson, Sr.
Howard A. Wolf
S. F. Wollmar
Charles H. Wurster

**ACTIVE MEMBERS.**

Harry Abel
R. C. Acton, Jr.
James R. Adams
Mrs. M. C. Adams
Hugh John Agnew
Walter Ainsworth
William G. Allan, R.P.E.
Chauncey D. Allen
George N. Anderman
Ronald A. Anderson
Charles F. Andrews, Jr.
Max Argent
John W. Armstrong, Jr.
Alma H. Arnold
Earle H. Ash
Ralph B. Ashenfelter
Paul H. Aspen
Samuel A. Ausker
Carlos Bacha
T. J. Bagley
Kenneth E. Baird
R. Bruce Baker
Frank B. Baldwin
William H. Balls
Barrie A. Barker
Dwight H. Barnes
L. Kingsley Barnes
Naaman F. Barr
Thurston W. Barrett
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William V. Bault
William C. Bechler, Jr.
Harold C. Beck
Lewis Beech
Henry L. Beidler
Robert J. Bennett
Samuel A. Benson
Carl B. Berger
Charles F. Berkey
John Berman
Edward Berry, Jr.
John T. Berry
N. L. Beyer
Harold B. Biehl
C. M. Biscoe
Herbert A. Bissout
Rodney Black
Earl C. Blaine
C. Gilbert Blatchley
Samuel Blumberg
Jacob Blumenthal
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E. W. Boehne
John P. Boland
Alfred J. Bolder
J. Harvey Bonine
Harry Boone
W. E. Borbonus
F. W. Borchers
A. G. Borowsky
William H. Boughter

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Thomas D. Bowes, M.E.
W. E. Bowler
William P. Bowron
Roy Martin Boyd
J. Frank Braceland
Edmund D. Brack
Joseph Bragin
Oscar Branch
J. G. Britton
William Brod
O. C. Brodersen
Eric C. Brodin
Brad Brodsky
George A. Brooke, Jr.
Raymond Brooks
F. Richard Brooks
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Col. Millard D. Brown
Nelson T. Brown
Harry R. Brownhill
Martin H. Buehler, 2nd
Louis Buek, Jr.
Roger W. Buntin
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W. E. Burkhard
Samuel K. Bushnell
Frank J. Butch
Richard H. Butler
Thomas M. Butterwich
Norman T. Buzby
C. B. Campbell
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James Carney
Harry D. Carson
Edward J. Casey
Howard H. Casey
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C. O. Caulton
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John S. Clement
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John Fernsler
Joseph T. Fewkes

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Ludwig Maier
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A. Homer Manwaring
Lee J. Markheim
Harry J. Markley
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J. Frederick Martin
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Richard Marx
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Hugh Matson
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John F. McClernon
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Robert McCreight, Jr.
R. R. D. McCullough
Leslie McKendrick
Archibald McLean, Jr.
John McLeister
D. Raymond McNeal
F. L. Meade, Jr.
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Wendel J. Meyer
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William H. H. Miller
T. E. Millership
George W. Mink, Jr.
Dr. L. John Minnick
Otto Mintzlaff
Allen R. Mitchell, III
Norman Mochel
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Harvey J. Moll
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Charles Frederic Moores
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H. Mouradian
Louis L. Moxey, III
Robert Moyer
Otto J. Mueller
Andrew C. Muir
John P. Murdoch
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Gilbert B. Mustin
Harold Mutch
A. Charles Myers
Joseph H. Myers
Willard G. Myers
Harry S. Nagin
H. C. Naschke, Jr.
D. Nast
Casper Neamand
William J. Nellis
Frank Nelson
Frank Nelson, Jr.
Paul G. Nelson
T. H. Nelson, Jr.
W. B. Nelson
George G. Nisbet
Albert Nesbitt
Theodore J. Nestopoulos

D. C. Nevins
Harry Newmark
John B. Nibling, Jr.
James H. Nicely
George William Nichols
Ezra K. Nicholson
J. T. Ninesteel
John W. Noble
Robert H. Norton
Edwin C. Nusbaum
William G. Oaks
Thomas O'Brien
A. M. Ornsteen, M.D.
J. L. Orr
Samuel Osterneck
L. E. Otte
Roland A. Otton
Charles W. Packard
Frank E. Paige
Leon E. Pamphilon
Joseph L. Parker
Ralph C. Parkes
H. E. Parkison
G. C. Parr
John C. Parry, Jr.
John A. Parsons
David E. Patterson
Michel B. Patterson
Henry N. Paul
Dorothy R. Pearlman
Joseph S. Pecker
Walter A. R. Pertuch
Gunnar Persson
Frank Perugini
Thomas Pesci
Robert P. Petersen
H. W. Pfeffer
Charles W. Pickard
Clyde G. Pinney
Lewen Pizor
Louis V. Place, Jr.
Joseph E. Podgor
F. Marvin Pollow
Hannibal E. Potter
Harold B. Pough
David A. Prentzel
Abraham Pritzert
Albert A. Pulley
Charles M. Putney
J. A. Queeney
Thomas J. Quinlan
A. M. Quinn
Jackson B. Raiguel
W. B. Rapp
H. P. Rassbach
C. H. Raub
J. H. Read
Miles H. Read
James K. Redding
George M. Reed
Albert V. Reese
Napoleon V. Reese
R. S. Reeve
H. R. Rehrig
James W. Reid, Jr.
William B. Reiff
Charles P. Renninger
F. X. Rettenmeyer
Frank J. De Rewal
Warren A. Reynolds
Fred Rhodes
David Richardson
Eugene Richardson
George Atwell Richardson
Max Riebenack, 3rd
R. Judson Riggs
Richard W. Rigler
R. G. Rincliffe
David P. Rind
Ralph W. Ritter
John S. Roberts
Adair Rogers
Doug Root
Benjamin Rose
Archie Rosenthal
John Rosser
Walter M. Rosskam
C. A. Rossman
Claude L. Roth
David Rothenberg
Joseph F. Ruchale
Cecilea B. Rudolph
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Andrew Ruppel
Charles E. Russell
Paul J. Rutan
Clinton H. Saffer
Herbert C. Sampter
Meyer Samson
Philip T. Samuel
Warren E. Sanborn
Matthew J. Scammell, Sr.
William F. Scanlin
Samuel Scatarige
George Schafenacker
Charles F. Schattler
Otto W. Schaum
George L. Schiel
C. E. Schley
Abraham Schmerling
Maurice Schmerling
William G. Schmidt
C. E. Schoble
Frederick Charles Scholler
Earl H. Scholl
Charles A. Schranz
Frederick G. Schranz
Edward A. Schuch
Carl L. Schuck
A. W. Schuller
Edwin A. Schuller
Axel Schulze
Gustav Schumacker, Jr.
Chris Schurr
Harry Schwarz
H. R. Scott
Theodore N. Seaman
Edward E. Sedgley
Raymond C. Seeburger, Jr.
Walter Seiler
W. B. Sewell
Samuel M. Shallcross
Ray G. Shankweiler
Jonathan Sharp
David T. Shaw
Kenneth Shaw
Robert F. Shaw
George B. Shearer
E. L. Shoemaker
Frederick Sier, Jr.
Samuel Silder
Julian S. Simsohn
Dolph Simons
Wendell F. Simmons

Henry A. Simpson
W. H. Sink
William M. Smelo
Clarence W. Smith
Edward D. Smith, Sr.
Frederick E. Smith
George D. Smith
H. J. Smith
J. Hywood Smith
Norman L. Smith
Norman L. Smith, Jr.
Richard H. Smith
Samuel C. Smith
Valentine G. Smith
Wilbur F. Smith
W. E. Soden
C. A. Spalding
Frank M. Speakman
Charles W. Spear
Woldemar Specht
Charles H. Speckman
Charles W. Speidel
W. Howard Stagg
C. J. Stahl
G. A. Stahl
Harold B. Staley
H. D. Steel
Carleton K. Steins
Constantine S. Stephano
Stephen Stephano
Arthur H. Stern
John Stern
N. Harper Steward
Douglas Stewart
Hugh T. Stewart
Thomas C. Stewart
E. G. Stoler
Gilbert Stone
William E. Storck
John F. Straitz
Bruce A. Stulginskis
George W. Supplee
E. K. Sweeney
Harold E. Sweeney
O. E. Szekely
H. D. Tallman
John S. Tawresey
B. W. Taylor
George C. Taylor
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George Y. Thomson
William M. Thomson
George H. Thornton
Henry H. Tomlinson
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Merrill A. Trainer
William C. Treston
Leonard A. Tucker
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William Valaitis
Harold C. Vance
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## LIBRARY NOTES.

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Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from nine o'clock A.M. until five o'clock P.M., Wednesdays and Thursdays from two until ten o'clock P.M.

## RECENT ADDITIONS.

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PEARSON, FRANK A., AND KENNETH R. BENNETT. Statistical Methods Applied to Agricultural Economics. 1942.

## NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

**Treatment of Pasteurella pestis Infection in Mice.**—R. LLOYD PHILLIPS and LUCY H. BARNES. We considered it of interest to test some of the sulfonamide drugs against a strain of *Pasteurella pestis* isolated in the United States. Many of the newer sulfonamide drugs have not as yet been tried against plague infection and we have therefore tested sulfathiazole (Merck and Co., Rahway, N. J.), sulfadiazine (Lederle Laboratories, New York), and succinyl sulfathiazole (Sharpe and Dohme, Philadelphia).

Reports of the trial of some sulfonamide drugs in experimental and clinical plague infection have been published in other countries. Schütze (*Lancet*, **1**: 266, 1939) has reported on the use of M. and B. 693 (sulfapyridine), soluseptasine and diaminodiphenylsulfone glucoside against the infection in rats and mice. M. and B. 693 was the most efficacious of the three drugs in both rats and mice, soluseptasine protected rats but not mice, and the sulfone protected mice but not rats. Antiplague serum produced results comparable to the results obtained with M. and B. 693. Sokhey and Dikshit (*Lancet*, **1**: 1040, 1940) found sulfathiazole superior to sulfapyridine and as effective as a good antiplague serum. Durand (*Archives de l'Institut Pasteur de Tunis*, **28**: 96, 1939) has reported Dagenon (sulfapyridine) effective against the infection in mice. Girard (*Bulletin de la Société de pathologie exotique*, **32**: 480, 1939) found sulfapyridine effective in mice and guinea pigs. Lastra, Gallardo and Videla (*La semana médica*, **1**: 1073, 1941) reported the use of sulfathiazole in conjunction with serum therapy in three human cases without beneficial effect. The three cases were said to be in grave circumstances when admitted for treatment. Wagle *et al.* (*Indian Medical Gazette*, **76**: 29, 1941) report the treatment of 237 cases of plague with antiplague serum, sulfapyridine and sulfathiazole. The case mortality was 28 per cent., 24 per cent. and 15 per cent. respectively as against 52 per cent. in controls (iodine

treated). Even when septicemia was present before treatment was begun mortality was reduced from 95 per cent. to about 50 per cent. in the treated cases. It was thought that a larger dose of the sulfonamide drugs than that used would reduce the case mortality still further.

The culture used for our experiment was a strain of *Pasteurella pestis* designated "Yreka," which Dr. K. F. Meyer of the Hooper Foundation, San Francisco, California, was so kind as to send to us. The inoculation was made subcutaneously in mice with 0.2 cc. of a 1–10,000 dilution of a 48 hour culture grown in Difco heart infusion broth. 0.2 cc. contained 800–1000 organisms as determined by blood agar counts. Although this inoculation was 100–1000 times the predetermined lethal dose, occasionally a mouse resisted this infective dose.

The mice used belonged to the dba strain which was the most susceptible to the infection of the four strains tested by us: a strain of inbred white mice obtained from a local dealer and C 57 Brown, C 57 Black and dba from the Roscoe P. Jackson Memorial Laboratory, Bar Harbor, Maine. The animals ranged in weight between 17 and 27 grams with the majority between 20 and 25 grams.

The mice were kept in metal cages, 10 mice per cage in a room prepared especially for these experiments. Extraordinary precautions were taken throughout for absolute isolation of these animals and the prevention of accidental communication of the disease. Though no mice were found infested at any time, precautions against flea infestation were taken at the time of isolation of the animals.

Cages were cleaned daily and all materials leaving the isolation room were autoclaved or incinerated.

The animals were treated by the drug-diet method, used extensively (J. T. Litchfield, Jr., *et al.*, *Journal of Pharmacology and Experimental Therapeutics*, **67**: 437, 1939; Raymond N. Bieter *et al.*, *Proceedings of the Society for Experimental Biology and Medicine*, **41**: 202, 1939 and *Journal of Pharmacology and Experimental Therapeutics*, **66**: P3, 1939) in comparative tests of sulfonamide drugs. This method was particularly advantageous in these experiments for its use eliminated handling of the infected animals for treatment.

Eight-tenths per cent. by weight of the drug was thoroughly mixed with finely ground fox chow pellets and placed in small metal feeding cups made from druggists' salve boxes. These cups were cylindrical, 2 inches in diameter, 3/4 inch deep. A hole 3/4 inch in diameter in the cover permitted entrance of only the animals' heads for feeding. The cups were placed in metal racks suspended about 3 inches above the bottom of the cage. These cups and racks eliminated, to a satisfactory extent, the difficulties of wastage and dirtying of food encountered with other feeding arrangements.

The cups each held 10 grams of food and enough cups were provided to supply the mice for 24 hours. After 24 hours the food was again weighed to determine the amount used. The drug consumed during the period could then be calculated for the group, but the drug ingested by individual mice is indicated only by the average for the group. Control mice were fed in the same way with finely ground pellets without the inclusion of the drug. Water was supplied in the usual type of water bottles.

The drug diet was given to Group A 2 days before infection and for 12 days after inoculation. For Group B, the drug diet was given at the time of inoculation and for 10 days afterward, because it was thought that inoculation of animals having an effective concentration of the drug already in the blood stream perhaps biased the results too greatly in favor of the efficiency of the drug. Longer delay in treatment was not attempted because control mice showed that food intake decreased rapidly as the infection developed so that insufficient drug intake would have resulted had treatment been delayed.

The mice were held for a period of 30 days after infection.

The results given in Table I show sulfathiazole and sulfadiazine to be very effective in the treatment of the infection in mice. Sulfadiazine may be more effective than sulfathiazole, for, even though the drug intake is less, the percentage of animals saved is higher with the sulfadiazine diet. The drug intake by animals of Group B was somewhat low for treatment but the percentage of animals saved by the sulfadiazine diet was still equal to that saved by sulfathiazole.

All animals of Group B, including controls, ingested less

TABLE I.

*The Effect of Sulfathiazole, Sulfadiazine and Succinyl Sulfathiazole on P. pestis in Mice.*

| Drug. | No. Mice. | Drug/Mouse Daily. | Deaths in Days. | | | | | | | | | Survival. | % Survival. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 3. | 4. | 5. | 6. | 7. | 9. | 12. | 13. | 23. | | |
| Group A | | | | | | | | | | | | | |
| Sulfathiazole | 19 | 40.8 mgms. | | 1 | 3 | 1 | | | | | | 14 | 73.6 |
| Sulfadiazine | 10 | 28.4 mgms. | | | | | | | | 1 | | 9 | 90.0 |
| Controls (No treatment) | 19 | — | 2 | 3 | 7 | 3 | 1 | | | 1 | | 2 | 10.5 |
| Group B | | | | | | | | | | | | | |
| Sulfadiazine | 20 | 19.8 mgms. | | | | | | 2 | 1 | | 1 | 16 | 80.0 |
| Succinyl Sulfathiazole | 20 | 14.7 mgms. | 5 | 6 | 3 | 3 | 1 | | | | | 2 | 10.0 |
| Controls (No treatment) | 20 | — | 3 | 8 | 5 | 2 | 1 | | | | | 1 | 5.0 |

Group A treated for two days before inoculation and for twelve days after. Group B treated at time of inoculation and for ten days after.

food than did Group A, probably because high temperature prevailed during the initial days of the experiment with Group B.

The ineffectiveness of succinyl sulfathiazole is probably due to the fact that it is absorbed poorly from the intestinal tract (A. D. Welsh, *Journal of Pharmacology and Experimental Therapeutics*, **75**: 231, 1942).

*Summary:* Sulfathiazole and sulfadiazine were effective in treating Pasteurella pestis infection in mice under the experimental conditions described. Succinyl sulfathiazole was ineffective.

**The Effect of pH on the Formation of Pyrrolidonecarboxylic Acid and Glutamic Acid during Enzymatic Hydrolysis of Glutathione by Rat Kidney Extract.**—GLADYS E. WOODWARD and FRANCIS E. REINHART (*Journal of Biological Chemistry*, **145**: 471, 1942). Both peptide bonds in GSH were found to be split by the enzymatic action of rat kidney extract in the pH range 5 to 9.5, as indicated by the liberation of cysteine. The rate was fastest in the range of pH 7 to 8.

Splitting of the γ-glutamyl peptide bond resulted in the formation of different proportions of glutamic and pyrrolidone-

carboxylic acids, depending upon the pH of the digestion mixture; glutamic acid predominated at pH values lower than about 6.6, and pyrrolidonecarboxylic acid in the more alkaline range.

The evidence indicated that pyrrolidonecarboxylic acid, as well as glutamic acid, was formed directly by the enzymatic action of the kidney extract on the γ-glutamyl bond.

**Studies on the Phosphorus Compounds of Brain. III. Determinations of Adenosine Triphosphate and its Decomposition Products in Fresh and Autolyzed Dog Brain.**—STANLEY E. KERR (*Journal of Biological Chemistry*, **145**: 647, 1942). Methods are described for analysis of the purines and the acid-hydrolyzable phosphorus attached to the nucleotide after separation of the nucleotides from the trichloroacetic acid filtrate by precipitation with mercuric acetate.

Dog brain frozen *in situ* with liquid air contains 19 mg. of acid-soluble purine nucleotide and 2 mg. of nucleoside and free purine nitrogen per 100 gm. Of the nucleotide nitrogen, 83.5 per cent. is adenine. Preliminary studies indicate that the remainder is guanine rather than hypoxanthine. The amount of acid-hydrolyzable phosphorus found attached to the nucleotide is close to that required for full phosphorylation.

During autolysis loss of acid-hydrolyzable phosphorus begins within 5 seconds, reaching a level corresponding to adenosine diphosphate in about 3 minutes, after which the nucleotide nitrogen slowly decreases to half the normal level within an hour, with a corresponding increase of nucleoside and free purine. No formation of inosinic acid is observed during 1 hour of autolysis. Adenine together with hypoxanthine and ribose is found in the nucleoside-free purine fraction, the adenine decreasing during autolysis.

The decomposition of nucleotide to nucleoside and free purine with a gradual decrease of adenine in the nucleoside fraction and no increase of inosinic acid suggests that adenylic acid in brain is first decomposed by a phosphatase rather than by a deaminase.

## BOOK REVIEWS.

INTERNAL COMBUSTION ENGINES, by J. A. Polson. Second edition, 548 pages, illustrations, 16 × 24 cms. New York, John Wiley & Sons, Inc. London, Chapman & Hall, Limited, 1942. Price $5.00.

There has been no cessation in the increase in popularity of the internal combustion engine. Greater efficiency is being obtained and many more applications are in order. This is for engines of the Otto cycle as well as of the Diesel cycle and when such possibilities as the gas turbine are considered, there seems to be no field free of the possibility of invasion. Latest information of a permanent and authoritative character is of utmost importance. It is with this in mind that the author has presented this second edition of his work.

This text is for mechanical engineering students who have had a thorough course in fundamental thermodynamics and who have a fair conception of the operation of internal combustion engines. The method used here is from the simple to the more complex. It begins with somewhat general statements on internal combustion engines and liquid and gaseous fuels. A chapter on combustion provides the opening for the treatment on thermodynamic relations of perfect gases with first constant, then variable specific heats. Various gas engines are next taken up including the Otto cycle engines, the Bogart gas engine, and blast furnace gas engines. With all this as a background, the automobile engine is discussed. Testing apparatus and calculation are thoroughly covered before the various parts of the automotive system are studied. These are carburetors, ignition systems which include such systems for airplane engines, and valves and timing. Certain factors affecting the design and performance of the airplane engine are considered separately, together with the characteristics of the engines. The Diesel engine is studied both from the standpoint of the large capacity engine and the engine of small capacity. This is followed by turbulence, scavenging, and supercharging; pre-combustion in chambers in Diesel engines; and Diesel fuel injection systems. The cooling of engines is treated in a separate chapter. The last chapter is devoted to the gas turbine, being mainly descriptive. There is an appendix containing useful tabular data including tables of energy and entropy for various gases.

The book gives a rather vigorous treatment. Much space is given over to drawings of the engines and parts. Characteristic curves are numerous. At the ends of some chapters there are problems for exercise and there is a subject index in the back of the book. For those who desire a modern and thorough coverage of the principles of internal combustion engines, this text can be recommended.

R. H. OPPERMANN.

FREQUENCY MODULATION, by August Hund. First edition, 375 pages, illustrations, 16 × 24 cms. New York and London, McGraw-Hill Book Company, Inc., 1942. Price $4.00.

The replacement of a signal current amplitude modulation with a modulation cycle of frequency variations has opened a new field in radio communications,

featured mainly by high fidelity transmission and avoidance of interference. As this new means of radio communication is further applied commercially there undoubtedly will be a need by many interested in radio to become acquainted with the engineering aspects of the subject. A thorough knowledge of the principles is essential to an appreciation of the design of the apparatus. This book presents those principles and shows their application in commercial apparatus.

Amplitude modulation concepts are firmly established. The author wisely begins here with a theoretical treatment of it and passes to phase and frequency modulation furnishing a comparison and evaluation of the three methods. There are discussed noise interference and wave propagation in the upper megacycle range of carrier frequencies. Application of Bessel Tables and Curves is demonstrated. Many principles are discussed in this early part of the book to prepare for the more practical treatments following. The next part, or chapter two, deals with auxiliary apparatus such as frequency modulators, frequency discriminators, and amplitude limiters. Much stress is laid upon showing that the speed of electrical actions in networks, partly due to their circuit elements, plays an important part in the best design of apparatus. This includes preaccentuators and deaccentuators. Chapter three gives descriptions, in terms of the results found in Chapter One, of all frequency modulation transmitters manufactured in this country. Chapter Four is devoted to receivers and Chapter Five deals with receiver and transmitter aerials as well as with feeders such as are being used in the range of frequencies assigned to frequency modulation stations. The appendix gives a detailed solution for the distribution of the modulation energy in the frequency spectrum; tables for the integral-sine, cosine and exponential functions; tables for circular, hyperbolic and exponential functions in terms of increasing arguments expressed in radians. There is also a list of references classified according to topics which should prove very helpful.

A feature of the book is the careful, detailed coverage, which is so necessary. It is up to date and should be valuable to the present understanding and future development of the subject.

R. H. OPPERMANN.

## PUBLICATIONS RECEIVED.

*The Theory of the Photographic Process*, by C. E. Kenneth Mees, D.Sc., F.R.S. 1124 pages, illustrations, 16 × 24 cms. New York, The Macmillan Company, 1942. Price $12.00.

*Workbook in Meteorology*, by Athelstan F. Spilhaus and James E. Miller. 163 pages, illustrations, 24 × 29 cms. New York and London, McGraw-Hill Book Company, Inc., 1942. Price $2.50.

*Water-White Hydrocarbons from Trinidad Asphalt*, by Halley Tansley Gaetz. First Report, 284 pages, tables and diagrams, 14 × 20 cms. Los Angeles, Grafton Publishing Corporation. Third Report, 136 pages, tables and diagrams, 14 × 20 cms. Los Angeles, Grafton Publishing Corporation. Plus Patent Applications and General Discussion.

*Science Remakes Our World*, by James Stokley. 298 pages, illustrations, 15 × 22 cms. New York, Ives Washburn, 1942. Price $3.50.

*A Start in Meteorology*, by Armand N. Spitz. 95 pages, illustrations, 14 × 20 cms. New York, The Norman W. Henley Publishing Company, 1942. Price $1.50.